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CONTENTS

DIFFERENTIALS IN TRUE NATURAL INCREASE. By P. K. Whelpton 233

VARIATE DIFFERENCE METHOD OF SEASONAL VARIATION. By Richard A. Robb 250

RANDOM EVENTS AND CYCLICAL OSCILLATIONS. By Simon Kuznets 258

SOME OBSERVATIONS CONCERNING THE ASSUMPTION OF NORMAL WORKING TIME. By H. LaRue Frain 276

MEASUREMENT OF THE DEMAND FOR FOOD. By Faith M. Williams 288

NOTES:

Geometric Mean of Percentages. By Arthur F. Burns 296

Use of Overlapping Quarterly Indexes in Erratic Series. By Donald S. Thompson and Robert W. Bachelor 301

Interlocking Memberships of Social Science Societies. By Stuart A. Rice and Morris Green 303

Financing the Nation's Trade and Industry (307); Progress of the Census (311); Unemployment Inquiry in the Fifteenth Census (314); Miscellaneous Notes (316); Members Added (327).

REVIEWS 330

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CONTENTS

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VARIATE DIFFERENCE METHOD OF SEASONAL VARIATION. By RICHARD A. ROBB 250

RANDOM EVENTS AND CYCLICAL OSCILLATIONS. By SIMON KUZNETS 258

SOME OBSERVATIONS CONCERNING THE ASSUMPTION OF NORMAL WORKING TIME. By H. LARUE FRAIN 276

MEASUREMENT OF THE DEMAND FOR FOOD. By FAITH M. WILLIAMS 288

NOTES:

GEOMETRIC MEAN OF PERCENTAGES. By ARTHUR F. BURNS 296

USE OF OVERLAPPING QUARTERLY INDEXES IN ERRATIC SERIES. By DONALD S. THOMPSON and ROBERT W. BACHELOR 301

INTERLOCKING MEMBERSHIPS OF SOCIAL SCIENCE SOCIETIES. By STUART A. RICE and MORRIS GREEN 303

FINANCING THE NATION'S TRADE AND INDUSTRY (307); PROGRESS OF THE CENSUS (311); UNEMPLOYMENT INQUIRY IN THE FIFTEENTH CENSUS (314); MISCELLANEOUS NOTES (316); MEMBERS ADDED (327).

REVIEWS: Donner: *Die Saisonschwankungen als Problem der Konjunkturforschung*, by N. J. Silberling (330); Kuczynski: *The Balance of Births and Deaths*, Vol. I, by A. J. Lotka (332); Bain: *Ores and Industry in the Far East*, by J. E. Orchard (334); Rose: *A Scientific Approach to Investment Management*, Fowler: *American Investment Trusts*, Steiner: *Investment Trusts, American Experience*, by W. W. Riefler (335); Grimes and Craigue: *Principles of Valuation*, by M. Sasuly (337); International Labour Office: *Methods of Compiling Housing Statistics*, by M. Leven (337); Seltzer: *A Financial History of the American Automobile Industry*, Epstein: *The Automobile Industry*, by W. L. Thorp (339); Schweizer: *Die Neue Goldwaehrung*, by R. Baumgartner (340); Watkins: *Bankers' Balances*, by M. G. Meyers (341); Kennedy: *Overhead Costs and the Shift to Machinery*, by H. Jerome (343); Kulp: *Casualty Insurance*, by R. H. Blanchard (344); Emmett: *The California and Hawaiian Sugar Refining Corporation*, by T. P. Blodgett (345); Guyot: *La Science Economique*, by F. H. Knight (345); Hamilton and Wright: *A Way of Order for Bituminous Coal*, by C. Goodrich (346).

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DIFFERENTIALS IN TRUE NATURAL INCREASE

By P. K. Whelpton, *Scripps Foundation for Research in Population Problems*

Differences in rates of population growth for various groups of people have been attracting much popular and scientific interest recently. The question of restricting immigration to the United States has led to comparisons of the natural increase among foreign and native elements of the population. The ancient race question now has a new aspect, for the large migration of Negroes to northern cities in the past decade has drawn attention to possible effects of an industrial environment and a cold climate on the natural increase of this race. Some persons are also wondering how the rapid movement of farm and village dwellers to larger cities will affect the population increase of the nation as a whole. Many comparisons of the growth of various groups of people have been made by students of population problems, birth-rates or ratios of children to women, and death-rates being used, both crude and standardized. The purpose in what follows is to apply to several groups the method of Dublin and Lotka [1] in the hope that the type of analysis which it provides may cast some further light on differentials in natural increase.

True rates for births, deaths, and natural increase calculated according to the Dublin and Lotka method have essential differences from crude and standardized rates which should be kept clearly in mind. Crude rates are based on the actual age and sex composition of the population group in question, which may have been much affected by migration and more influenced by the specific birth- and death-rates of previous decades than of those of the period under consideration. If the current specific birth- and death-rates of a population group are applied to some standard million to secure standardized rates, this

[1] Louis I. Dublin and Alfred J. Lotka, "On the True Rate of Natural Increase," this Journal, Vol. XX, September, 1925.

statement may be carried one step further, for not only may the age and sex composition of this standard million have been affected by migration, but it probably will bear *no* relation to the specific birth- and death-rates of the group being studied. For true rates, however, a stable age and sex composition is calculated on the assumption that the given specific rates remain in operation indefinitely, and that no migration occurs. Each population group thus has its own particular age and sex composition, directly related to its own specific rates and used as a basis for computing its true rates. Because of these peculiarities, true rates give a different picture of the situation in population groups than do crude or standardized rates. Furthermore, they give a much more accurate measure of the power of natural increase inherent in the population.

To use the Dublin and Lotka method it is necessary to have birth-rates by age of mother, and 1_x values from life tables showing the number of survivors at certain ages throughout the life span. The main sources of these data for recent years in the United States are *Abridged Life Tables 1919–1920*, and current volumes of *Birth Statistics*, both of which cover certain states and cities. Life tables for other groups of the population have been prepared by insurance companies and other agencies, but little material regarding birth-rates at various ages is available from such sources. Since the birth-rate usually is more variable than the death-rate, the choice of population groups to be compared here is largely limited to those included in *Birth Statistics*. Seventeen groups were finally selected for study as the best available to represent native white, foreign white, and Negro sections of the population both in cities and rural parts and in industrial and agricultural states.

CRUDE AND TRUE RATES COMPARED

True rates for births, deaths, and natural increase show the rôle played in population growth by the majority of the groups in question to be quite different from that indicated by crude rates. Although each of the seventeen groups for which data are presented in Table I has some natural increase according to the crude rates for 1920, the true rates indicate decreases in seven instances. This change from an increase to a decrease is most marked for native whites in Washington, D. C., where the *crude* rate of increase is 6.7 per 1,000 but the *true* rate is *minus* 14.2. A decline almost as great is shown by Negroes in twelve large cities from 2.8 per 1,000 to *minus* 12.7. Five other groups change from a plus to a minus rate, while three more show a decline of over 50 per cent. Only native whites in Kentucky, North Carolina and South

Carolina, and in rural United States have high true rates as well as crude. Foreign whites in Kansas, Minnesota and Wisconsin have a true rate much above the crude, but their crude rate is very low since the heavy immigration to these states went on before 1910 resulting in an age composition by 1920 unfavorable for a high birth-rate and low death-rate. Barring this single exception true natural increase is less than crude; much less in most cases.

Negroes in seven southern states, who might be expected to stand at the top in natural increase come fifth with a true rate only about two-thirds that of native whites in three of these states. These southern Negroes, however, stand much higher than Negroes in sixteen northern states or in twelve large cities (of which all but three are northern cities), these last two groups showing decreases. But in spite of these decreases, the true natural increase of all Negroes in the United States compares favorably with that of all native whites because over 85 per cent of the former live in the South where the rate in all likelihood is about as high as for the seven states shown here. It is interesting to speculate on how many more years a large northward migration of Negroes would have to continue in order to lower the Negro rate of true natural increase to the native white level.

Rates for three foreign white groups are shown in Table I to allow certain comparisons among themselves and with native whites. In applying the Dublin and Lotka method, however, it is assumed that no migration now occurs. A foreign white group would thus become native in one generation and perhaps should be referred to as a group having the specific birth- and death-rates of foreign-born whites in 1920. But using the shorter term for brevity, foreign whites have a true rate of natural increase 19.6 above that of native whites in Connecticut, Massachusetts and New York, and the eleven large cities, and 11.0 above in Kansas, Minnesota and Wisconsin. Certain native groups exceed the foreign, however, native whites in Kentucky, North Carolina and South Carolina being 0.9 above foreign whites in Kansas, Minnesota and Wisconsin, and native whites in rural United States having a larger margin over foreign whites in Connecticut, Massachusetts and New York, and the large cities.

It is interesting to compare foreign whites in Kansas, Minnesota and Wisconsin with those in the other two areas from the standpoint of country of origin. About 70 per cent of the foreign whites in these north central states came from northwestern Europe, Germany and English Canada as compared with about 40 per cent in the three eastern states and eleven cities. These last two groups drew nearly 60 per cent of their immigrants from such countries as Poland, Austria, Rus-

sia, Italy and French Canada which have had birth-rates considerably higher than the other countries above mentioned. Judging from the country of origin of the foreign born, the true rate of natural increase should be higher in Connecticut, Massachusetts and New York, and the eleven large cities than in Kansas, Minnesota and Wisconsin. Evidently some force has been at work lowering the rate of population increase to a greater extent among the foreign whites in these cities and industrial states than among those in the more rural area.

This difference between true natural increase in urban and rural communities, also hinted at in the case of Negroes, merits special consideration. Invariably within the race and nativity classification and usually in general, the rural groups have much larger true rates of natural increase than the urban groups. Native whites in Kentucky, North Carolina and South Carolina (78 per cent rural) have the largest true rate of increase in Table I. Foreign whites in Kansas, Minnesota and Wisconsin (50 per cent rural) are second, native whites in rural United States are third and Negroes in seven southern states (70 per cent rural) are fifth. Foreign whites in Connecticut, Massachusetts and New York (92 per cent urban) come fourth, the highest urban group, but well below the three rural groups. At the lower extreme all the groups are urban—Negroes in twelve large cities and sixteen northern states (86 per cent urban) and native whites in Washington, D. C., in eleven large cities, and in Connecticut, Massachusetts and New York (81 per cent urban) all of which show true natural decreases.

In general there seems to be no question but that true rates of increase under urban conditions are much lower than under rural conditions. The only exceptions of note in Table I are foreign whites in eleven large cities, and in Connecticut, Massachusetts and New York (92 per cent urban), both groups having fairly high true rates of natural increase. As previously mentioned, the majority of the foreign born in these areas come as adults from such high birth-rate countries as Italy, Poland and Russia, and apparently the women bear a large number of children, much as they would have done in their native land. Urbanization seems to have some influence even in this generation, however, for these groups stand below the more rural foreign whites in Kansas, Minnesota and Wisconsin even though the majority in this group come from countries of lower birth-rate.

It is in the second generation that the influence of urbanization is really seen. Immigrants from high birth-rate countries have children born and reared in the United States who seem to be like the children of native parents in their response to urban or rural living conditions. As an illustration, nearly half of the native white women in Connecticut,

Massachusetts and New York, and in eleven large cities have foreign born mothers.[1] It is possible that the specific birth-rates of native women of foreign mothers are twice as high as those of native women of native mothers though other evidence indicates a much smaller difference.[2] But even if it were the case, the true rate of natural increase for the three states would be only 5 per 1,000, and that for the cities would be even lower. Both these groups would thus stand much below the rural groups. It appears, therefore, that the high rates for the two urban foreign groups in Table I do not indicate any permanent failure of urbanization to lower natural increase, but simply that there may be a lag of one generation in this process when a large proportion of the population are immigrants from high birth-rate countries.

Perhaps a clearer idea of the significance of differences in the true rate of increase for the various groups may be had by considering the result if each rate remained in effect for one hundred years. On this basis native whites in Kentucky, North Carolina and South Carolina would have a gain in population of 636 per cent in a century, and in rural United States a gain of 456 per cent. At the other extreme native whites in Washington, D. C., and Negroes in twelve large cities would be only about one-fourth as numerous at the end of a century as at the beginning, the percentage declines being 76 and 72 respectively. A stationary population is well represented by native whites in California, Oregon and Washington, the change here in a century being a decline of only one per cent. These data together with those for the other groups are shown in Table I.

CRUDE AND STABLE AGE AND SEX COMPOSITION

Since the rate of natural increase for each area is the difference between its birth-rate and death-rate, the impulse may be to examine these last rates in accounting for variations in the former. For all whites in Connecticut, Massachusetts and New York, for example, the difference between the crude rate of natural increase of 9.3 for the 1920 population and the true rate of 1.9 for the stable population [3] might appear to be due about as much to the rise in the death-rate from 13.7 to 17.6 as to the fall in the birth-rate from 23.0 to 19.5.

It must be remembered, however, that by definition the same specific birth-rates are used for calculating both the crude and true birth-rates of any group and similarly the same specific death-rates for both the crude and true death-rates. A difference between the crude and true

[1] The exact figures are 45.52 per cent and 47.97 per cent respectively.

[2] Warren S. Thompson, *The Ratio of Children to Women in the United States, 1920*, Chapter V and *passim* (in manuscript).

[3] Stable population, *i. e.* having a stable age and sex composition, *infra* p. 234.

birth-rates or death-rates of any group, therefore, must be due to differences in the age and sex composition. Actually, the age composition of the stable population does differ significantly from that of the 1920

TABLE I

CRUDE AND TRUE RATES PER 1,000 PERSONS FOR BIRTHS, DEATHS, AND NATURAL INCREASE OF VARIOUS POPULATION GROUPS

	Birth-Rate		Death-Rate		Rate of Natural Increase		Per cent gain in 100 years
	Crude 1920 (1)	True per year (2)	Crude 1920 (3)	True per year (4)	Crude 1920 (5)	True per year (6)	(7)
Native whites:							
Conn., Mass., N. Y.	16.8	14.6	12.8	20.6	4.0	—6.0	—45
Ky., N. C., S. C.	33.3	32.0	11.1	11.8	22.2	20.2	636
Kans., Minn., Wis.	25.3	22.1	8.9	13.8	16.4	8.3	129
Calif., Ore., Wash.	20.0	17.1	10.9	17.2	9.1	—.1	—1
Eleven large cities (8)	15.8	14.1	12.1	22.0	3.7	—7.9	—55
Washington, D. C.	18.3	9.8	11.6	24.0	6.7	—14.2	—76
Urban United States	18.8	15.2	12.7	20.8	6.1	—5.6	—43
Rural " "	28.2	29.3	10.7	12.0	17.5	17.3	456
Foreign whites:							
Conn., Mass., N. Y.	39.3	28.1	16.1	14.5	23.2	13.6	275
Kans., Minn., Wis.	22.1	30.3	20.6	11.0	1.5	19.3	576
Eleven large cities (8)	42.5	27.3	16.2	15.6	26.3	11.7	220
All whites:							
Conn., Mass., N. Y.	23.0	19.5	13.7	17.6	9.3	1.9	22
Kans., Minn., Wis.	25.1	23.1	11.0	13.5	14.1	9.6	161
Eleven large cities (8)	23.8	19.1	13.4	18.7	10.4	.4	4
Negroes:							
Seven southern states (9)	35.2	31.3	17.6	18.3	17.6	13.0	264
Sixteen northern states (10)	26.0	20.7	21.0	25.5	5.0	—4.8	—38
Twelve large cities (11)	24.5	16.8	21.7	29.5	2.8	—12.7	—72

(1) Adjusted births from Elbertie, Foudray, *U. S. Abridged Life Tables 1919–1920*, pp. 58, 60, allocated to native and foreign mothers according to *Birth Statistics, 1920*, Table 3. See *supra* Appendix, for urban and rural groups.

(2) Calculated from formulae (41) and (47), Louis I. Dublin and Alfred J. Lotka, "On the True Rate of Natural Increase," this JOURNAL, Vol. XX, September, 1925, pp. 335–337. Also *supra* Table III and Appendix.

(3) Deaths from *Mortality Statistics 1920*, Table 4, divided by July 1, 1920 populations, estimated by arithmetical method. The urban-rural division is here made at places of 10,000 and over.

(4) Birth-rate minus rate of natural increase.

(5) Birth-rate minus death-rate.

(6) Calculated from formula (24), Dublin and Lotka, *op. cit.*, p. 332. See also *supra* Table III and Appendix.

(7) Calculated from the true rate of natural increase by the compound interest formula.

(8) Baltimore, Boston, Buffalo, Cleveland, Detroit, Los Angeles, New York City, Philadelphia, Pittsburgh, San Francisco, Washington, D. C., Foudray, *op. cit.*, p. 12. Chicago, New Orleans and St. Louis were excluded because not in the 1920 Birth Registration Area.

(9) Maryland, Virginia, North Carolina, South Carolina, Kentucky, Tennessee and Missouri, in each of which Negroes constituted more than 5 per cent of the population in 1920, Foudray, *op. cit.*, p. 12. Missouri is not a southern state, but it has little influence on the group since it contained but 5.2 per cent of the group's Negro population in 1920.

(10) Connecticut, Massachusetts, New York, New Jersey, Pennsylvania, Ohio, Indiana, Illinois, Michigan, Wisconsin, Minnesota, Kansas, Utah, Washington, Oregon and California, in each of which Negroes constituted less than 4 per cent of the population in 1920, Foudray, *ibid.*

(11) Baltimore, Boston, Buffalo, Chicago, Cleveland, Detroit, New Orleans, New York, Philadelphia, Pittsburgh, St. Louis and Washington, D. C., Foudray, *ibid.*

population in most of the groups studied as is shown in Table II.[1] To illustrate with all whites in Connecticut, Massachusetts and New York, in the 1920 population 9.8 per cent of the females are under five years of age, 26.1 per cent between five and nineteen years, 46.8 per cent between twenty and forty-nine years, and 17.3 per cent fifty years or over, while in the stable population the proportions are 8.6 per cent, 24.3 per cent, 41.7 per cent, and 25.4 per cent respectively. With such differences in female age distribution, it is natural that the same specific death-rates give a general death-rate of 13.7 for the 1920 population as a whole against 17.6 for the stable population, and that the same specific birth-rates give a general birth-rate of 23.0 for the 1920 population against 19.5 for the stable population. One of the most striking differences in female age composition in Table II is found among Negroes in twelve cities, the 20–49 age period amounting to 61.2 per cent and the 50 and over period to 9.9 per cent of the 1920 population, but to 44.5 per cent and 26.5 per cent respectively of the stable population. Since the other two age periods are about equal in both populations, the general death-rate based on identical specific death-rates shows a large divergence, being 21.7 in the 1920 population as against 29.4 in the stable population. Similarly, the general birth-rate based on identical specific birth-rates falls from 24.5 in the 1920 population to 16.8 in the stable population. Variations between the general birth-rates and death-rates of the 1920 and stable populations of other groups in Table I are also due in large measure to differences in age composition.

Only data on the age composition of females have been cited so far, partly because the reproductive period is more limited than in the case of males and hence the birth-rate of the population as a whole is more affected by variations in the age composition of the female portion of the population. However, the age composition of one sex is so nearly like that of the other in both the 1920 and stable populations of each area that what has been said about the females will apply also to the males.

Variations in the sex composition of the 1920 and stable populations are of far less importance than those in age composition in causing differences between the crude and true birth-rates and death-rates obtained from identical specific rates. In twelve of the groups dealt with here, the differences between the number of males per 1,000 females in the 1920 and stable populations do not exceed 5 per cent and hence have negligible influence. For certain groups the variation is larger,

[1] The differences would be much greater in the foreign white group if the 1920 population were classified by nativity instead of by nativity of mother as explained in Note (2) of Table II. For this reason, the crude rates for births, deaths, and natural increase of native and foreign whites in Table I do not correspond exactly with the 1920 age distribution of these groups in Table II.

however, foreign whites in Kansas, Minnesota and Wisconsin having 1,025 males per 1,000 females in the stable population as against 1,313 in the 1920 population. Similar ratios for foreign whites in eleven large cities are 983 against 1,108, and for Negroes in sixteen northern states 966 against 1,062. The tendency in these cases is to raise the true birth-rate and lower the true death-rate compared with the crude birth- and death-rates as the number of males relative to females declines to the stable population level. Near the other extreme are native whites in Washington, D. C., the stable population containing 1,006 males per 1,000 females but the 1920 population only 848. A similar situation occurs with Negroes in seven southern states, the ratios being 1,076 and 977. The tendency in these cases is to lower the true birth-rate and raise the true death-rate relative to the crude birth- and death-rates as the number of males relative to females increases to the stable population level.

The age and sex composition of the stable population differs from that of the 1920 population in each area in part because of the elimination of immigration, and also because of the holding of the specific birth-rates at a stationary level instead of allowing them to continue to decline as they had been doing previous to 1920. The net effect of these two factors is to increase the proportion of the population in the younger adult age periods. This has been well set forth by Dublin and Lotka for the total white population of the United States [1] and will not be discussed here.

Since various areas within the United States are compared with each other in this paper, the additional factor of internal migration must be considered. To illustrate its importance, the 1920 census shows that 1,646,000 white persons born in Connecticut, Massachusetts and New York were living in other states compared with 1,069,000 white persons born in other states but living in these three, a net loss of 577,000. Kentucky, North Carolina and South Carolina sustained a similar net loss of 695,000 white persons. On the other hand California, Oregon and Washington gained 2,020,000 white persons in this manner, an amount which equaled 47 per cent of the native population of these states in 1920. The extent of migration in city populations would doubtless be much more striking if it could be measured. Unfortunately Washington, D. C., is the only city for which the census gives the necessary data. Of the native white population in this city, 182,000 persons were born elsewhere, while only 50,000 persons born in Washington, D. C., were living in other parts of the United States. This net gain of 132,000 amounted to 44 per cent of the native white

[1] Dublin and Lotka, *op. cit.*, pp. 314–316.

population of the city. Among the Negro groups, the seven southern states had sent out 585,000 Negroes more than they received, and the sixteen northern states had received 669,000 more than they had sent out. This latter number amounted to 52 per cent of the 1920 Negro population of these states.

If the age composition of these migrants was about the same as that of the non-migrating natives, age composition and crude or true birth- and death-rates would be little affected by the movement. While exact data are lacking on this question of age, the probability is that in most cases young adults are relatively more numerous in the migrating than in the non-migrating portion of the population.[1] On this basis, internal migration tends to lower the proportion of the population in the young adult age periods in Connecticut, Massachusetts and New York where the natives sent out exceed those received. It is opposed to the previously mentioned factors of immigration and a declining birth-rate in past decades, both of which tend to increase the proportion of young adults in this area. Internal migration is the main factor at work in Kentucky, North Carolina and South Carolina, for few immigrants have gone to this area, and the specific birth-rates have undoubtedly undergone less decline here than in most other areas. All three factors work together for the most part in the Pacific States; for they have many foreign-born inhabitants, the specific birth-rates have no doubt declined considerably, and the gain through internal migration has been very heavy. When nativity groups are considered, it should be remembered that internal migration as herein defined does not apply to the foreign white groups, neither does immigration apply to the native white groups. In most cases, the decline in the birth-rate in the country of origin of the foreign born has been much less than that in the United States, so this factor affects the foreign-born groups only to a minor degree.

VARIATIONS IN STABLE AGE COMPOSITION

Comparisons between the age composition of the stable populations shown in Table II are more interesting in certain respects than those between the 1920 and stable populations within an area. For one thing, more striking variations may be found, as illustrated by contrasting the native whites of Connecticut, Massachusetts and New York with those of Kentucky, North Carolina and South Carolina. In the former area 6.6 per cent are under five years of age, 20.2 per cent are five to nineteen years, 41.4 per cent are twenty to forty-nine years,

[1] The movement to California, Oregon and Washington might be an exception, due to older adults retiring to California in particular.

TABLE II

PER CENT DISTRIBUTION BY SPECIFIED AGE PERIODS OF THE FEMALES IN THE 1920 AND STABLE POPULATIONS (1)

	1920 Population (2)				Stable Population (3)			
	0–4	5–19	20–49	50 and over	0–4	5–19	20–49	50 and over
Native whites:								
Conn., Mass., N. Y.	11.5	28.3	43.0	17.2	6.6	20.2	41.4	31.8
Ky., N. C., S. C.	13.2	34.7	39.2	12.9	14.0	33.4	38.2	14.4
Kans., Minn., Wis.	15.5	37.6	37.3	9.6	9.9	26.8	40.2	23.1
Cal., Ore., Wash.	10.6	29.5	46.6	13.3	7.8	22.8	40.6	28.8
11 large cities	11.8	28.4	47.5	12.3	6.3	19.7	41.6	32.4
Washington, D. C.	6.9	20.8	56.4	15.9	4.6	15.4	39.0	41.0
Urban U. S.	11.2	29.4	45.9	13.5	6.9	20.8	41.5	30.8
Rural U. S.	13.6	35.7	37.5	13.2	12.8	31.5	38.9	16.8
Foreign whites:								
Conn., Mass., N. Y.	8.7	24.6	49.3	17.4	11.9	30.2	40.3	17.6
Kans., Minn., Wis.	5.2	20.9	50.1	23.8	13.4	32.3	38.4	15.9
11 large cities	8.6	24.1	50.6	16.7	11.5	29.6	40.8	18.1
All whites:								
Conn., Mass., N. Y.	9.8	26.1	46.8	17.3	8.6	24.3	41.7	25.4
Kans., Minn., Wis.	11.1	30.4	42.8	15.7	10.3	27.5	40.1	22.1
11 large cities	9.8	25.7	49.5	15.0	8.3	23.9	42.0	25.8
Negroes:								
7 Southern States	11.9	36.0	41.7	10.4	13.6	34.1	39.2	13.1
16 Northern States	8.0	23.4	57.5	11.1	8.6	25.1	43.4	22.9
12 large cities	6.5	22.4	61.2	9.9	7.0	22.0	44.5	26.5

(1) See Table I for limits of certain areas.

(2) *Fourteenth Census*, Vol. II, Chap. III. Native-born daughters of foreign-born mothers are omitted from the native group and included in the foreign group to lessen the distorting effect of immigration on the 1920 age distribution and also because the stable population includes descendants of foreign mothers as explained in the text. See Appendix for allocation to native and foreign mothers of daughters of mixed parentage.

(3) Calculated from formula (1) Dublin and Lotka, *op. cit.*, p. 329. See also *supra* Appendix.

and 31.8 per cent are fifty years or over while in the latter area the corresponding figures are 14.0 per cent, 33.4 per cent, 38.2 per cent, and 14.4 per cent. This rural southern area has nearly twice as large a proportion of persons under twenty as the urban northern area and scarcely half as large a proportion older than fifty. Differences nearly as startling exist between the stable Negro populations of the southern states and the twelve large cities.

Causes of variations in the age composition of the different stable population groups are quite distinct from those responsible for variations between the 1920 and stable populations of the same area, which have been discussed previously. No two stable populations have exactly the same specific birth- and death-rates; it is the difference here that affects the age composition. Table III gives these data in detail. For example, the death-rate of whites at age 0 [1] varies from a low of 59.1 per 1,000 in California, Oregon and Washington to a high of 87.1 in urban United States, and that of Negroes from 99.4 in the seven southern states to 140.6 in the twelve cities. At age twenty-seven,

[1] During the first year after birth.

TABLE III

SPECIFIC BIRTH- AND DEATH-RATES AND EXPECTATION OF LIFE (1)

	Female Births per 1,000 Women Aged (2)									Birth-Rate per 1,000 women 15–44 (3)	Death-Rate per 1,000 Persons at Exact Age (4)			Expectation of life at exact age 0 (4)
	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54		0	27	57	
Native whites:														
Conn., Mass., N. Y.	x	13.2	55.4	60.5	43.2	24.7	8.9	.6	x	71.4	84.9	6.2	22.0	54.8
Ky., N. C., S. C.	.2	38.6	103.8	103.0	85.8	66.3	29.7	3.2	.1	148.9	66.0	6.4	16.8	57.5
Kans., Minn., Wis.	x	17.0	72.9	81.1	64.3	44.2	18.4	2.3	.1	103.2	63.9	5.4	15.6	59.9
Cal., Ore., Wash.	.1	24.6	70.6	62.7	42.7	24.9	8.8	.8	x	81.9	59.1	6.5	17.4	58.3
11 large cities	x	12.7	53.3	58.2	41.6	23.8	8.6	.6	x	68.6	85.8	6.3	25.2	53.3
Washington, D. C.	.3	21.1	38.1	40.2	33.2	19.4	4.9	.7	x	54.4	70.3	5.7	21.8	56.8
Urban U. S.	x	12.1	53.2	60.3	46.5	29.2	10.7	1.0	x	73.4	87.1	6.7	23.8	52.2
Rural U. S.	.1	29.4	93.9	97.7	80.1	59.6	26.4	3.2	.1	133.9	65.1	5.6	14.4	60.6
Foreign whites:														
Conn., Mass., N. Y.	x	23.3	91.9	97.6	77.4	50.2	18.9	1.9	x	124.2	84.9	6.2	22.0	54.8
Kans., Minn., Wis.	x	29.2	108.3	99.7	80.3	59.2	27.0	4.5	x	139.8	63.9	5.4	15.6	59.9
11 large cities	x	22.5	88.4	94.0	74.5	48.3	18.2	1.8	x	119.4	85.8	6.3	25.1	53.3
All whites:														
Conn., Mass., N. Y.	x	14.8	65.1	73.7	56.9	35.7	13.1	1.2	x	89.6	84.9	6.2	22.0	54.8
Kans., Minn., Wis.	x	17.5	75.0	83.0	66.7	47.2	20.5	2.9	.1	107.2	63.9	5.4	15.6	59.9
11 large cities	x	14.3	63.1	71.4	55.1	34.6	12.7	1.2	x	86.7	85.8	6.3	25.1	53.3
Negroes:														
7 Southern States	1.0	52.3	102.5	91.8	74.5	54.7	22.2	4.3	.3	139.7	99.4	13.6	30.3	45.9
16 Northern States	1.2	55.5	75.3	54.3	37.7	23.6	9.6	1.4	.1	90.4	128.4	11.6	38.0	41.6
12 large cities	1.3	49.5	64.6	43.3	32.2	19.5	6.1	1.0	x	76.8	140.6	11.8	45.5	39.5

x indicates less than .1 per thousand.
(1) See Table I for limits of certain areas.
(2) See Appendix.
(3) The age composition of the women 15–44 in each group being that of a stationary population based on *United States Abridged Life Tables 1919–20*, pp. 18–19. Both male and female births are included. See also Appendix.
(4) See Appendix.

there is much less variation, either relative or absolute, but at age fifty-seven the relative differences are greater. Here the low white rate is 14.4 in rural United States and the high rate 25.2 in the eleven cities, while for Negroes the seven southern states are low at 30.3 and the twelve cities high at 45.5. These specific death-rates at different ages may be expressed most conveniently as a single item by the expectation of life.[1] Negroes in the twelve cities have the shortest expectation of life, 39.5 years, while the native whites in rural United States have the longest, 60.6 years, a gain of about 53 per cent.

The specific female birth-rates in Table III are more variable than the specific death-rates, as would be expected. The female birth-rate for women 20–24 years of age ranges from 38.1 per 1,000 for the native white population of Washington, D. C., to 108.3 for foreign whites in Kansas, Minnesota and Wisconsin, an increase of about 185 per cent. This and the next age periods are the two of highest fertility and show less relative variation than most others. At age 35–39 the female birth-rate in the same areas varies from 19.4 to 59.2, an increase of 205 per cent, and at age 40–44 the increase is over 450 per cent. The best single item for picturing the birth-rate is total births per 1,000 women 15–44 whose age composition within these limits is that of a stationary population. This is shown for each group in Table III rising from 54.4 for native whites in Washington, D. C., and 68.6 for native whites in eleven cities to as much as 139.8 for foreign whites in Kansas, Minnesota and Wisconsin, and 148.9 for native whites in Kentucky, North Carolina and South Carolina. The highest rate for these groups represents an increase of about 175 per cent over the lowest rate, a much greater variation than that of 53 per cent in the death-rate as measured by the expectation of life.

There is a very close direct relation between the birth-rate per 1,000 women 15–44 in Table III and the per cent of the stable population 0–4 in Table II as might be expected. In fact if the birth-rate of Negro women in seven southern states was 139.9 instead of 139.7, the ranking would be identical in both series. It might be thought that a similar direct relation would exist between a high expectation of life and the per cent of the stable population fifty years of age and over. This is not the case, however. Native whites in rural United States who stand at the top in expectation of life are fourth from the bottom in per cent of persons fifty and over, while Negroes in twelve large cities who have the shortest expectation of life stand sixth in per cent of per-

[1] The expectation of life is not a standardized death-rate, simply being 1,000 divided by the death-rate per 1,000 of a stationary population. Ordinarily there is much less variation in the age composition of different stationary populations than of actual populations or those stabilized according to the Dublin and Lotka method.

sons fifty and over. The more important factor in determining the per cent in the oldest age group appears to be the spread between the birth-rate and the death-rate. The larger the birth-rate in comparison with the death-rate, the more rapid the growth of population and the smaller the proportion of persons over fifty. Thus, the ranking of the groups in respect to true rate of natural increase [1] in Table I is very nearly the same as the ranking in respect to per cent fifty years of age and over in Table II.

In conclusion, the preceding discussion may be summarized as follows:

(1) The Dublin and Lotka method is valuable for analyzing the natural increase of population groups for which certain basic data on specific birth- and death-rates are available.

(2) The true rate of natural increase of each population group here studied is below the crude in sixteen out of seventeen cases, usually much below and frequently a minus quantity. It is much higher in rural than in urban areas, in agricultural than in industrial states, and is lowest of all in large cities. Northern Negroes have a much lower true rate of increase than southern, though all Negroes probably stand ahead of all whites because a much larger proportion live in a more rural environment. Foreign whites stand ahead of native whites in the same area, though below native whites in certain more rural areas.

(3) The 1920 age composition is much different from the stable age composition in most groups largely because the former is affected by immigration, internal migration, and a declining birth-rate. All three factors work together in the Pacific States to maintain at a high figure the proportion in the young adult ages chiefly at the expense of the older ages. Internal migration partially offsets the other two in Connecticut, Massachusetts and New York in this respect, and more than offsets them in Kentucky, North Carolina and South Carolina.

(4) The stable age composition varies markedly between groups, because of differences in specific birth- and death-rates. The specific birth-rates are much more variable than the specific death-rates. Natives whites and Negroes in certain southern states, foreign whites in Kansas, Minnesota and Wisconsin, and native whites in rural United States have the highest specific birth-rates, while native whites and Negroes in cities and industrial states have the lowest.

APPENDIX

SPECIFIC BIRTH-RATES

In computing the specific female birth-rates appearing in Table III (the $m(a)$ values of Dublin and Lotka) the number of women by five

[1] Which is the excess of the birth-rate over the death-rate.

year age periods was obtained directly from the *14th Census*, Vol. II, pages 188–311, for each population group studied except urban and rural native whites. For these two groups the age periods "20–44" and "45 and over" from page 371 were divided into five year periods along a smoothed curve.

The number of female births by age and nativity of mother for the native, foreign-born, and all white populations of each group of states and of Washington, D. C., was obtained from *Birth Statistics 1920*, Table 5, births at "age unknown" being distributed proportionally, together with unregistered births as indicated by *United States Abridged Life Tables 1919–20*, Table 17. Dividing the number of births by the number of women gave the rates used.

In the other groups studied, female births by age of mother are not reported in the form desired, hence certain estimates were necessary. For the native, foreign, and all white populations of eleven large cities, the number of births by nativity of mother was obtained from *Birth Statistics 1920*, Table 3, and the number of female births by applying sex ratios calculated from Table 2. A proportional increase was made to allow for unregistered births as indicated by *United States Abridged Life Tables 1919–20*, Table 17. As births in these cities are not reported by age of mother, it was assumed that the specific female birth-rates at various ages in the native white population of this group of cities bore the same relation to each other as those of native whites in Connecticut, Massachusetts and New York where 83 per cent of native white women 15–44 live in urban communities. Similar assumptions were made for the foreign and all white populations in these cities. Rates thus secured give the correct number of births when applied to the 1920 population and it is believed that only slight differences from the results of exact rates would occur even if applied to a population of greatly different age composition.

A like process was followed for Negroes in twelve large cities, the specific female birth-rates at various ages being assumed to bear the same relation to each other as those of Washington, D. C., and New York State (79 per cent of whose Negroes live in New York City and Buffalo). For Negroes in seven southern states, the number of female births was obtained from *United States Abridged Life Tables 1919–20*, Table 17, and the specific female birth-rates were assumed to bear the same ratio to each other as those calculated for Maryland, Virginia, North Carolina, South Carolina and Kentucky using births by age of mother from *Birth Statistics 1920*, Table 5. Negroes in sixteen northern states were similarly treated.

For the urban and rural groups, the total white births in the United

States were estimated to be 2,498,800 on the assumptions that the registered white births in *Birth Statistics 1920*, page 44, represented 93 per cent of the births in the Registration Area, and that births in the United States bore the same relation to births in the Registration Area as did children 0–1, January 1, 1920. This total was distributed among urban and rural native and foreign mothers based on the distribution of children 0–1, January 1, 1920, shown in *14th Census*, Vol. II, page 371, and special tabulations furnished by the Bureau of the Census to the Scripps Foundation for a Monograph, *The Ratio of Children to Women in the United States in 1920*, with slight corrections for a higher infant mortality among urban infants than rural, and among infants of foreign mothers than of native mothers.

As urban and rural births are not reported by age of mother, it was assumed that the specific birth-rates at various ages in the native white urban population bore the same relation to each other as those of native whites in Connecticut, Massachusetts and New York (81.2 per cent urban) and in the native white rural population as those of native whites in Kansas, Minnesota and Wisconsin, and Kentucky, North Carolina and South Carolina (each group weighted equally, 68.8 per cent rural).

SPECIFIC DEATH-RATES

The specific death-rates and expectation of life values for all whites from *United States Abridged Life Tables 1919–20* are used for native whites and foreign whites in Table III *infra*. Similarly, the number of survivors at specified ages for all whites (the $p(a)$ values of Dublin and Lotka, or the 1_x function of Life Tables 5 and 6 but with a radix of unity) are used in computing the true rates for births, deaths, and natural increase of native and foreign whites in Table I *infra*. This procedure is accurate for native whites in Kentucky, North Carolina and South Carolina where over 99 per cent of all whites are native born, and very nearly so for native whites in Washington, D. C., who make up over 92 per cent of all whites as a whole and over 85 per cent of all whites in each five year age period under sixty years.

In Kansas, Minnesota and Wisconsin the white population in certain age periods is so largely native born, while in others the death-rates in Table IV vary so little with nativity that there probably is little error in using the all white life table values for the native white. Foreign-born death-rates in Table IV differ more from all white rates in these states, particularly in the younger age divisions where a difference is more serious. This is partly due to the small numbers of foreign born on which the rates are based and to their peculiar age distribution.

TABLE IV

SPECIFIC DEATH-RATES OF WHITE FEMALES IN CERTAIN STATES AND PER CENT NATIVE BORN (1)

	Conn., Mass., N. Y.				Kans., Minn., Wis.			
	Specific Death-Rates per 1,000			Per cent native born	Specific Death-Rates per 1,000			Per cent native born
	Native born	Foreign born	All white		Native born	Foreign born	All white	
0–4	25.8	24.6	25.8	99.4	17.8	14.5	17.8	99.7
5–9	2.9	3.9	2.9	97.0	2.4	3.4	2.4	98.8
10–14	2.0	2.4	2.0	92.7	1.9	2.8	1.9	97.5
15–19	3.2	3.3	3.2	84.9	2.9	4.1	2.9	95.9
20–24	4.8	4.9	4.8	73.3	4.8	5.8	4.8	94.0
25–29	6.2	5.6	6.0	64.4	6.0	6.2	6.0	90.0
30–34	6.6	6.4	6.6	60.0	6.4	8.2	6.6	84.7
35–39	6.7	6.8	6.8	56.8	6.4	6.3	6.4	79.7
40–44	7.9	7.6	7.8	57.8	6.6	7.1	6.7	75.6
45–49	10.1	10.6	10.3	58.3	7.5	7.8	7.6	72.9
50–54	13.7	15.2	14.3	57.3	10.7	10.5	10.7	66.0
55–59	19.4	23.2	21.0	58.7	15.3	16.0	15.6	61.2
60–64	27.7	34.3	30.2	61.5	22.4	21.8	22.1	59.6
65–69	43.0	54.9	47.7	60.8	33.7	31.5	32.7	52.8
70–74	63.6	82.8	71.4	59.4	54.1	53.8	53.9	46.6
75–79	96.7	112.3	102.8	61.2	86.2	85.6	85.8	42.7
80–84	150.0	162.0	154.5	62.7	139.0	136.9	137.8	40.6
85–89	222.3	218.6	220.9	64.1	187.5	205.0	198.0	39.8
90–94	310.9	291.2	303.9	64.2	257.1	308.7	289.2	37.8

(1) Number of female deaths from *Mortality Statistics 1920*, Table 4. Female population from *Fourteenth Census*, Vol. II, pp. 188–285.

Perhaps a better indication of the situation may be had by including children of foreign or mixed parentage [1] with those foreign born. The 0–4 rate is now 18.3, the 5–9 rate 2.6, the 10–14 rate 2.0, etc., all much nearer the all white rates though somewhat above them. But even if death-rates of foreign whites are as much as 10 per cent larger than those of all whites, which appears an extreme assumption, the true rate of natural increase would only drop from 19.3 to 18.8, while the true birth-rate would only rise from 30.1 to 30.2 and the true death-rate from 10.8 to 11.4, changes too small to affect the preceding discussion significantly.

In Connecticut, Massachusetts and New York, the native white specific death-rates in Table IV are very slightly below the all white rates up to age 50–54 and somewhat more so in the older ages, so the true death-rate in Table I may be as much as .2 too high and the true rate of natural increase .2 too low. The foreign white specific death-rates in these states probably are larger than the all white rates in nearly every age division, for when children of foreign and mixed parentage are included the 0–4 death-rate is 26.7, the 5–9 rate 3.0, and

[1] Deaths of persons of foreign parentage are not separated from those of mixed parentage in *Mortality Statistics*.

the 10–14 rate 2.0. The true death-rate for this group in Table I may therefore be as much as .4 too low, and the true rate of natural increase .4 too high. None of the changes would have any appreciable effect on the standing of the native and foreign groups in this area or the discussion regarding them. This statement can also be made for native whites in California, Oregon and Washington. Deaths by age and nativity are not published in *Mortality Statistics* for individual cities, so no check was made for native whites in eleven large cities. It is possible that this group would be similar to Connecticut, Massachusetts and New York in this respect.

For native whites in urban and rural United States, the death-rates and expectation of life values in Table III *infra*, and the number of survivors at specified ages used in computing the true rates in Table I are based on the aggregate white data in the 1919–20 Life Tables and the aggregate, urban and rural white data in *United States Life Tables 1890, 1901 and 1910*, the ratio of urban and rural death-rates to aggregate death-rates in 1910 being assumed to apply in 1920. No adjustment was made for variation between native white and all white rates, so the true death-rates for urban and rural United States in Table I may be as much as .2 or .3 too high and the true rates of natural increase .2 or .3 too low.

It may seem unreasonable that a 10 per cent increase of specific death-rates of foreign whites in Kansas, Minnesota and Wisconsin would only result in the true death-rate increasing from 10.8 to 11.4 or 5.6 per cent. As previously mentioned, however, different specific rates result in a population having a different age distribution. It is this change in age composition that prevents true death-rates from varying in the same ratio as specific death-rates.

THE VARIATE DIFFERENCE METHOD OF SEASONAL VARIATION

By Richard A. Robb, *University of Glasgow, Scotland*

In recent issues of this Journal there have appeared articles in which first differences have been employed to measure seasonal variation. It seemed, however, that their use was more or less empirical, and it is proposed in this paper to discuss their theoretical significance.

Let Y = original item,

U = corresponding ordinate of trend,

$1+S$ = corresponding seasonal index,

$1+W$ = corresponding cyclical index,

and ϵ = corresponding accidental index,

so that we have

$$Y = U(1+W)(1+S)(1+\epsilon) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

It should be noted that S represents the deviation of the seasonal index about unity, and similarly for the others.

If (1) were expanded we should have

$$Y = U+UW+S(U+UW)+\epsilon U(1+W+S\overline{1+W}),$$

that is, we assume the original item is made up of the following four components:

(1) Trend;

(2) Cyclical, which depends on (1);

(3) Seasonal, which depends on the sum of (1) and (2);

(4) Accidental, which depends on the sum of (1), (2), and (3).

Formula (1) is, therefore, a reasonable hypothesis regarding the constitution of Y.

We may write formula (1) as follows:

$$Y = U(1+S+W+WS)(1+\epsilon)$$

or

$$Y' = 1+S+W+\epsilon+\epsilon(S+W)+SW+\epsilon WS \ldots\ldots\ldots\ldots (2)$$

where

$$Y' = \frac{Y}{U}.$$

Now S is a variation with a period of one year and hence may be represented by

$$S = \sum_{r=1}^{6} a_r \cos\left(\frac{2\pi}{12}x \cdot r + a_r\right) \text{ where } a_6 = 0, \; x \text{ in months,}$$

the different harmonics corresponding to periods of 12, 6, 4, 3, $\frac{12}{5}$ and 2 months.

We do not know any period for W, or whether there is any period, but it is sufficient for our needs to express it as a sum of cosines, where the periods are taken as two years or more.

For example,

$$W = b_{24}\cos\left(\frac{2\pi}{24}x+\beta_{24}\right)+b_{25}\cos\left(\frac{2\pi}{25}x+\beta_{25}\right)$$
$$+\ldots+b_k\cos\left(\frac{2\pi}{K}+\beta_k\right)+\ldots K=24,\ 25\ldots\ldots$$

Taking differences on both sides of formula (1) or (2), we have $\Delta^r Y' = \Delta^r S+\Delta^r W+\Delta^r\epsilon+\Delta^r K$, where $K=\epsilon(S+W+SW)+SW$, $r=1$, $2,\ldots\ldots$

In this paper we shall consider only second differences, *i.e.* $r=2$, giving

$$\Delta^2 Y' = \Delta^2 S+\Delta^2 W+\Delta^2\epsilon+\Delta^2 K.$$

We have, then, for $\Delta^2 S$,

$$\Delta^2\left\{a_1\cos\left(\frac{2\pi}{12}x+a_1\right)\right\} = a_1\left(2\sin\frac{2\pi}{24}\right)^2\cos\left(\frac{2\pi}{12}x+a_1+\frac{7\pi}{6}\right)$$
$$=\frac{a_1}{3.73}\cos\left(\frac{2\pi}{12}x+a_1+\frac{7\pi}{6}\right)$$

$$\Delta^2\left\{a_2\cos\left(\frac{\pi}{3}x+a_2\right)\right\} = a_2\cos\left(\frac{\pi}{3}x+a_2+\frac{4\pi}{3}\right)\ldots\ldots\ldots\ldots\ldots(3)$$

$$\Delta^2\left\{a_3\cos\left(\frac{\pi}{2}x+a_3\right)\right\} = 2a_3\cos\left(\frac{\pi}{2}x+a_3+\frac{3\pi}{2}\right)$$

$$\Delta^2\left\{a_4\cos\left(\frac{2\pi}{3}x+a_4\right)\right\} = 3a_4\cos\left(\frac{2\pi}{3}x+a_4+\frac{5\pi}{3}\right)$$

$$\Delta^2\left\{a_5\cos\left(\frac{5\pi}{6}x+a_5\right)\right\} = 3.73a_5\cos\left(\frac{5\pi}{6}x+a_5+\frac{11\pi}{6}\right)$$

$$\Delta^2\left\{a_6\cos(\pi x+a_6)\right\} = 4.00\,a_6\cos(\pi x+a_6+2\pi).$$

For W we find, for example,

$$\Delta_2\left(b_{24}\cos\frac{2\pi}{24}x+\beta_{24}\right) = \frac{b_{24}}{7.66}\cos\left(\frac{2\pi}{24}x+\beta_{24}+\frac{13}{12}\pi\right)$$

$$\Delta_2\left(b_{36}\cos\frac{2\pi}{36}x+\beta_{36}\right) = \frac{b_{36}}{11.47}\cos\left(\frac{2\pi}{36}x+\beta_{36}+\frac{19\pi}{18}\right)$$

$$\Delta_2\left(b_{48}\cos\frac{2\pi}{48}x+\beta_{48}\right) = \frac{b_{48}}{15.29}\cos\left(\frac{2\pi}{48}x+\beta_{48}+\frac{25\pi}{24}\right)\text{ and so on.}$$

If we assume that the original items have the standard deviation of errors σ, we can easily see that the second differences have a standard deviation of 2.45σ.

By differencing, therefore, we increase except in the first two cases the amplitudes of the seasonal harmonics, and decrease those of the cycle to a very small value. It appeared feasible, then, to take the second differences of Y' and apply Fourier Analysis, taking one year as the period. In $\Delta^2 Y'$ we see (1) cycle effects will be small, as explained above; (2) as regards the $\Delta^2 K$ terms, where $K=\epsilon(S+W+SW)+SW$, the terms multiplied by ϵ will again yield accidental variations, which will only affect the seasonal harmonics in that there will be a slightly larger probable error, and in any case the magnitude of such terms is always small; when S is multiplied by W, as for example $\left(a_1 \cos \frac{2\pi}{12}x\right)\times\left(b_1 \cos \frac{2\pi}{24}x\right)$ we obtain products which again can be expressed as a sum of cosine terms whose periods will not be any of the periods of the seasonal harmonics *viz.* 12, 6, ... months; hence the second differences of SW will appear as accidental variations of small magnitude, without altering the seasonal harmonics except as to probable error. The standard deviation of errors is, of course, greater but this is balanced by the increase in the amplitude of the seasonal harmonics.

The Fourier Analysis of $\Delta^2 Y'$ with the period of one year will yield the best value of $\Delta^2 S$. Having found the various harmonics of $\Delta^2 S$ as given by (3) we easily find the corresponding harmonics of S and hence the value of S for the various months of the year.

To illustrate the method I shall take the data concerning the United States storage holdings of butter. This series was selected by Dr. Bauman in applying his thirteen-months-ratio-first-difference method,[1] and as I wish to discuss this method later, I have chosen the same series for applying the scheme outlined above. The cycle variations in this series are small compared with the seasonal but another example will be given later where cycle variations are roughly six times as large as the seasonal.

The data are given in Table I and Chart I of Dr. Bauman's paper and need not be reproduced here. The trend is given by

$$U=119.0\,x+50{,}327$$

the unit being 1000 pounds; and $x=1, 2, \ldots$. measured from January 1916. The values of $Y'=\frac{Y}{U}$ are given in Table I.

[1] A. O. Bauman, this JOURNAL, September 1928, pp. 282–290.

CHART I

FLUCTUATIONS OF PIG-IRON PRODUCTION

ABOVE AND BELOW LINE OF TREND 1904–1914

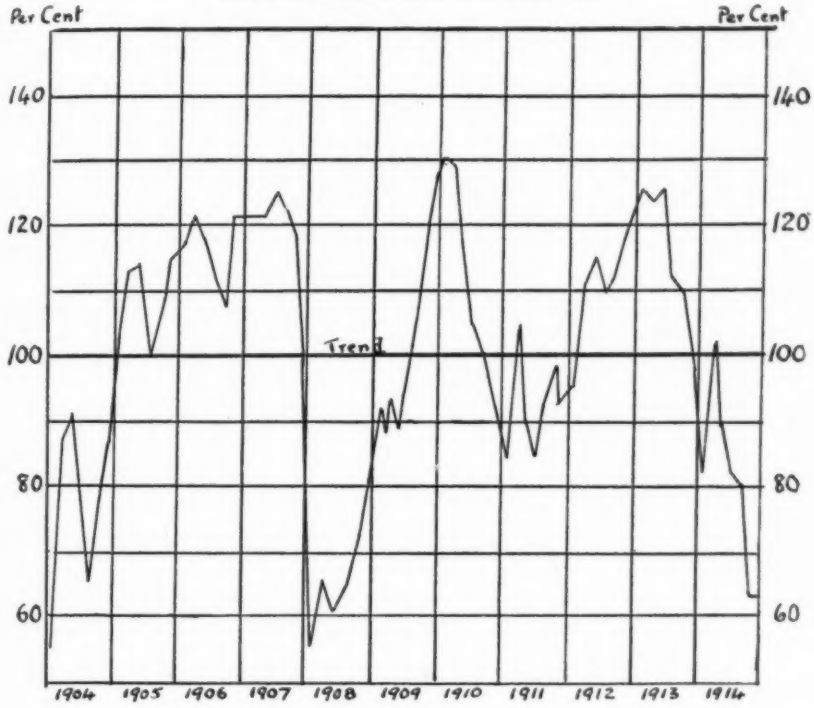


TABLE I

ORIGINAL DATA DIVIDED BY TREND IN PER CENT (Y′)

	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928
Dec. '15 97.3													
Jan.	61.7	58.7	49.9	67.2	68.3	72.0	59.4	26.7	24.6	72.3	60.8	27.2	31.9
Feb.	29.7	32.6	35.2	44.1	40.1	47.0	38.2	14.7	15.9	45.4	40.6	12.2	21.3
Mar.	6.6	13.1	27.3	21.7	22.3	25.5	15.4	7.9	12.6	17.1	26.8	4.6	....
Apr.	2.1	6.9	17.8	17.5	13.4	13.3	6.5	5.3	14.3	5.9	26.9	5.2	....
May	13.8	19.0	23.6	53.3	22.7	37.3	22.2	16.6	35.8	20.4	46.9	38.0	....
June	105.5	95.3	91.2	163.0	92.6	106.5	113.1	102.8	118.8	99.7	133.0	135.0	....
July	200.4	169.2	163.5	222.8	178.4	142.1	172.7	166.4	214.3	170.4	200.4	217.1	....
Aug.	206.4	205.2	183.5	236.5	202.8	158.0	187.2	167.7	249.5	200.2	210.7	243.4	....
Sept.	195.6	206.6	162.0	218.8	198.5	153.9	161.2	156.6	244.3	177.7	190.8	219.7	....
Oct.	165.5	189.1	148.7	180.1	177.8	132.9	122.9	124.3	214.5	147.5	153.3	176.7	....
Nov.	130.3	150.6	119.5	131.7	139.1	110.8	79.4	83.6	159.9	115.9	97.7	123.6	....
Dec.	89.1	95.4	80.4	95.9	102.1	82.2	44.5	49.1	104.0	81.7	52.0	68.6	....

Table II gives the second differences of Y', *i.e.* the first item will be February '16 − twice January '16 + December '15. = 3.6. (Data being that of Table I.)

TABLE II

$\Delta^2 Y'$ (PER CENT)

	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	Average
Dec. '15 3.6													
Jan.	8.9	6.6	6.4	0.7	10.4	3.5	— 1.6	5.2	5.4	— 1.4	6.4	7.4	4.8
Feb.	18.6	13.3	— 1.6	18.2	8.9	9.3	13.9	4.2	5.0	17.1	13.9	8.2	10.8
Mar.	16.2	18.3	15.3	40.0	18.2	36.2	24.6	13.9	19.8	25.7	19.9	32.2	23.4
Apr.	80.0	64.2	61.8	73.9	60.6	45.2	75.2	74.9	61.5	64.8	66.1	64.2	66.0
May	3.2	— 2.4	4.7	—49.9	15.9	—33.6	—31.3	—22.6	12.5	— 8.6	—18.7	—14.9	—12.1
June	—88.9	—37.9	—52.3	—46.1	—61.4	—19.7	—45.1	—62.3	—60.3	—40.9	—58.1	—55.8	—52.4
July	—16.8	—34.6	—41.5	—31.4	—28.7	—20.0	—40.5	—12.4	—40.4	—52.3	—29.2	—50.0	—33.2
Aug.	—19.3	—18.9	8.2	—21.0	—16.4	—16.9	—12.3	—21.2	—24.6	— 7.7	—17.6	—19.3	—15.6
Sept.	— 5.1	—21.0	—15.9	— 9.7	—18.0	— 1.1	— 5.2	— 8.4	—24.8	— 1.4	—18.1	—10.1	—11.6
Oct.	— 6.0	—16.7	— 9.9	12.6	1.7	— 6.5	8.6	6.2	— 1.3	— 2.6	9.9	— 1.9	— 0.5
Nov.	10.8	9.7	25.9	8.2	6.9	5.8	17.1	10.0	24.2	13.3	20.9	18.3	14.3
Dec.	4.3	31.2	— 9.9	— 0.6	5.1	1.6	5.8	15.8	4.8	0.7	9.8		6.0

When $\Delta^2 Y'$ is analyzed employing a period of one year we find, using the figures of the last column of Table II,

$$\Delta^2 Y' = 25.6 \cos(\theta - 52°8') + 22.4 \cos(2\theta - 220°39') + 16.9 \cos(3\theta - 346°37') + 10.4 \cos(4\theta - 146°46') + 7.2 \cos(5\theta - 246°16') + 2.4 \cos 6\theta,$$

where $$\theta = \frac{2\pi}{12}x = \frac{\pi}{6}x.$$

Therefore

$$S = 95.5 \cos(\theta - 262°8') + 22.4 \cos(2\theta - 100°39') + 8.5 \cos(3\theta - 256°37') + 3.4 \cos(4\theta - 86°46') + 1.9 \cos(5\theta - 216°16') + 0.6 \cos(6\theta) = \sum_{r=1}^{6} A_r, \text{ say.}$$

Hence giving θ the values 0°, 30°, 60°, ... , 360°, we find S for the various months as shown in Table III.

TABLE III

SEASONAL VARIATION COEFFICIENTS (PER CENT)

	A_1	A_2	A_3	A_4	A_5	A_6	Total
Dec.	—13.2	— 4.2	—2.0	0.2	—1.5	0.6	—20.1
Jan.	—58.6	17.0	—8.3	2.8	0.8	—0.6	—46.9
Feb.	—88.5	21.1	2.0	—3.0	0.2	0.6	—67.6
Mar.	—94.6	4.2	8.3	0.2	—1.1	—0.6	—83.6
Apr.	—75.3	—17.0	—2.0	2.8	1.7	0.6	—89.2
May	—35.9	—21.1	—8.3	—3.0	—1.9	—0.6	—70.8
June	13.2	— 4.2	2.0	0.2	1.5	0.6	13.3
July	58.6	17.0	8.3	2.8	—0.8	—0.6	85.3
Aug.	88.5	21.1	—2.0	—3.0	—0.2	0.6	105.0
Sept.	94.6	4.2	—8.3	0.2	1.1	—0.6	91.2
Oct.	75.3	—17.0	2.0	2.8	—1.7	0.6	62.0
Nov.	35.9	—21.1	8.3	—3.0	1.9	—0.6	21.4

The seasonal indexes are obtained by adding 100 per cent to each of the numbers in the last column. It will be noticed that the first row of figures in Table III is for December; the reason for this is that the first item in the original data was December, 1915.

In Table IV I give the seasonal indexes together with those obtained by the link-relative and the ratio-difference method of Dr. Bauman.

TABLE IV

SEASONAL INDEXES

Method	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Ratio-First-Difference.....	56.1	34.5	17.2	12.1	28.4	109.7	184.2	204.0	190.2	160.2	122.3	81.1
Link-Relative....	51.9	32.6	16.6	10.2	29.4	115.3	189.3	208.4	191.3	158.1	119.1	77.8
Variate-Difference.....	53.1	32.4	16.4	10.8	29.2	113.3	185.3	205.0	191.2	162.0	121.4	79.9

It will be seen that the new method favors the link-relative but not to any great extent.

The other example I have chosen is the monthly tonnage of pig iron produced in the United States from 1904–1914.[1] These data were selected because there are large cycle movements as well as many irregular fluctuations. In Chart I, I reproduce the values of Y', and in Table V, I compare the seasonal indexes obtained by (1) link-relative method, (2) new method.

TABLE V

Method	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Link-Relative...	100.7	94.6	106.9	102.6	102.7	95.2	94.1	97.1	98.1	104.5	101.2	102
Variate-Difference..	98.4	94.7	106.9	103.8	103.4	96.7	95.9	98.1	99.2	105.2	99.6	98

The agreement is satisfactory except for January and December. This is due mostly to the fact that there is one item in December for which the link-relative is 67, and the Fourier Analysis attributes more significance to this item as seasonal than the link-relative method.

We shall now examine Dr. Bauman's method. It will be recalled that his first step is to "relate in percentages the actuals of each year to the monthly average or a selected average or median of the year" taking, in his example given above, the average for the year. This average is a rough approximation to trend and cyclic factors, and in dividing the actuals by such there is a certain elimination of these

[1] *Review of Economic Statistics*, prel. Vol. I, p. 66; or E. E. Day *Statistical Analysis*, p. 243.

factors; the resultant ratios can then be favorably compared for seasonal variation although accidental variations will be not so well eliminated as by the use of link relatives. Having obtained first differences it is necessary to find some suitable average for January in order that these differences may be cumulated to yield seasonal indexes. The median of the above mentioned ratios for all the Januaries is taken. Now if this method is to be satisfactory it must yield similar results if we make some other month, say June, our starting point and take as our average for the year the figures from June to May inclusive. I therefore carried through the necessary computations as given in Table VI, which corresponds to that of Table II of Dr. Bauman's.

TABLE VI

A. RATIOS TO AVERAGE OF THE YEAR (JUNE TO MAY INCLUSIVE)

	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926
Average for Year.....	52,436	55,780	52,211	65,723	61,321	50,211	47,479	48,822	76,831	64,081	61,584
June.....	102.7	89.6	94.1	137.2	85.7	123.5	141.7	128.6	96.6	99.4	141.1
July.....	195.5	159.5	169.1	188.0	165.4	165.0	216.8	208.5	174.6	170.2	213.0
Aug.....	201.8	193.9	190.3	199.9	188.4	183.8	235.5	210.4	203.6	200.4	224.3
Sept.....	191.7	195.7	168.3	185.3	184.9	179.5	203.2	196.9	199.8	178.2	203.5
Oct.....	162.6	179.5	154.9	152.9	166.0	155.3	155.2	156.6	175.7	148.1	163.8
Nov.....	128.3	143.3	124.7	112.1	130.1	129.7	100.4	105.5	131.2	116.7	104.5
Dec.....	88.0	90.9	84.1	81.8	95.7	96.4	56.4	62.1	85.5	82.4	55.8
Jan.....	58.1	47.7	70.4	58.4	67.7	69.8	33.9	31.2	59.5	61.5	29.2
Feb.....	32.3	33.7	46.3	34.3	44.2	45.0	18.7	20.2	37.5	41.1	13.1
Mar.....	13.0	26.2	22.8	19.1	24.0	18.1	10.1	16.1	14.2	27.1	4.9
Apr.....	6.9	17.1	18.5	11.5	12.6	7.6	6.8	18.3	4.9	27.4	5.6
May.....	19.0	22.8	56.4	19.6	35.4	26.3	21.3	45.8	17.0	47.7	41.1
June.....	95.3	88.1	172.7	79.9	101.1	134.3	131.9	151.9	82.9	135.6	146.3

B. FIRST DIFFERENCES

	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
1916.....	102.7	92.8	6.3	—10.1	—29.1	—34.3	—40.3	—29.9	—25.8	—19.3	— 6.1	12.1	76.3
1917.....	89.6	69.9	34.4	1.8	—16.2	—36.2	—52.4	—43.2	—14.0	—17.5	— 9.1	5.7	65.3
1918.....	94.1	75.0	21.2	—22.0	—13.4	—30.2	—40.6	—13.7	—24.1	—23.5	— 4.3	37.9	116.3
1919.....	137.2	50.8	11.9	—14.6	—32.4	—40.8	—30.3	—23.4	—24.1	—15.2	— 7.6	8.1	60.3
1920.....	85.7	79.7	23.0	— 3.5	—18.9	—35.9	—34.4	—28.0	—23.5	—20.2	—11.4	22.8	65.7
1921.....	123.5	41.5	18.8	— 4.3	—24.2	—25.6	—33.3	—26.6	—24.8	—26.9	—10.5	18.7	108.0
1922.....	141.7	75.1	18.7	—32.3	—48.0	—54.8	—44.0	—22.5	—15.2	— 8.6	— 3.3	14.5	110.6
1923.....	128.6	79.9	1.9	—13.5	—40.3	—51.1	—43.4	—30.9	—11.0	— 4.1	2.2	27.5	106.1
1924.....	96.6	78.0	29.0	— 3.8	—24.1	—44.5	—45.7	—26.0	—22.0	—23.3	— 9.3	12.1	65.9
1925.....	99.4	70.8	30.2	—22.2	—30.1	—31.4	—34.3	—20.9	—20.4	—14.0	0.3	20.3	87.9
1926.....	141.1	71.9	11.3	—20.8	—39.7	—59.3	—48.7	—26.6	—16.1	— 8.2	0.7	35.5	105.2
A, Average (Median) First Difference.....	102.7	75.0	18.8	—13.5	—29.1	—36.2	—40.6	—26.6	—22.0	—15.2	— 6.1	18.7	87.9
B, Cumulative First Differences.....	102.7	177.7	196.5	183.0	153.9	117.7	77.1	50.5	28.5	13.3	7.2	25.9	113.8
C, Trend.....	0.0	0.9	1.8	2.8	3.7	4.6	5.5	6.4	7.4	8.3	9.2	10.1	11.1
D (B–C)	102.7	176.8	194.7	180.2	150.2	113.1	71.6	44.1	21.1	5.0	— 2.0	15.8	102.7
D[1].....	112.7	186.8	204.7	190.2	160.2	123.1	81.6	54.1	31.1	15.0	8.0	25.8	.....
E, Seasonal Variation...	113.3	187.8	205.8	191.3	161.1	123.8	82.1	54.4	31.3	15.1	8.0	25.9	.....
Taking January as starting point.....	109.7	184.2	204.0	190.2	160.2	122.1	81.1	56.1	34.5	17.2	12.1	28.4	.....

It will be noticed in line D that we are faced with the difficulty of a negative expression for April. To remove this I have selected

another value for June such that the total for all the months would be approximately 1,200. This method can only at best be an approximation, but it is not easy to find a satisfactory solution. Having obtained our seasonal indexes, we see that there is an appreciable difference in some months when they are compared with those obtained with January as our starting point, as given in the last line of Table VI. The negative expression for April could also have been removed by selecting as the average for each column in Table VI the mean of the three middle items instead of the median. This would give June a value of 108.5 but the results obtained show a bigger divergence from the last line of Table VI than by the method previously outlined. The whole question would seem to be concerned with a proper choice of average for the initial month from which first differences are cumulated. As regards the bias due to trend, it is obvious that Dr. Bauman's method will reduce this to some extent, but there is still considerable distortion as evidenced by the 11.1 per cent obtained in Table VI, whereas the amount of trend as measured by the method of least squares is 2.4 per cent.

RANDOM EVENTS AND CYCLICAL OSCILLATIONS[1]

By Simon Kuznets, *National Bureau of Economic Research, Inc.*

That the method of cumulation leads sometimes to extraordinary results is pointed out by Messrs. C. J. Bullock, W. M. Persons and W. L. Crum in their reply to Karl Karsten's "The Harvard Business Indexes—A New Interpretation."[2] They say: ". . . It is quite possible for a very disordered succession of positive and negative items when used as a generating series to yield a cumulative series in which the oscillatory fluctuations partake of the wave-like form encountered in . . . economic movements" (page 82). The authors have demonstrated their point by showing how numbers derived in an apparently irrelevant and casual fashion, when summated, result in a line of clear-cut cyclical oscillations.

These results were in no way accidental. It is not only "possible" that a summation of a random series will yield cycles, but also quite certain that this will be the case. Indeed this is the first thesis of a remarkable memoir by Professor Eugen Slutsky, "The Summation of Random Causes as the Source of Cyclic Processes," published in 1927 in *The Problems of Economic Conditions*, Moscow, Vol. III, No. 1.[3] In this article Professor Slutsky develops in great detail and with a considerable mathematical foundation two theses: (a) "That cyclic, *i. e.* undulatory processes may originate owing to a summation of the mutually independent chance causes; and (b) that these chance waves may show a certain regularity, being an imitation, in a lesser or greater degree, of the strictly periodical fluctuations."

The exposition which follows deals only with the first thesis which is well established and which reveals implications of considerable significance to the statistical and theoretical approach to time series analysis. Without attempting to develop fully the mathematical groundwork underlying this first thesis, I shall present in the simplest form its content and basis, and then proceed to elucidate its implications.

I

The raw materials used by Professor Slutsky are numbers derived from tabulations of drawings in a state lottery in the U. S. S. R.

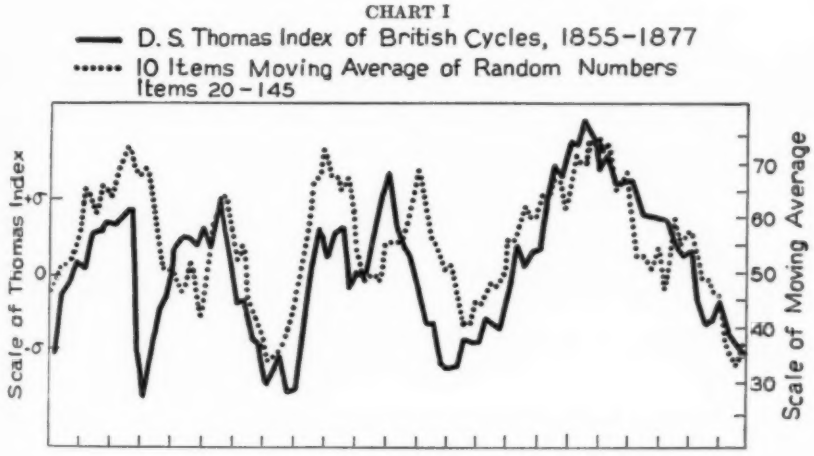
[1] The author expresses his gratitude to Professor Henry Schultz of the University of Chicago for valuable suggestions, and to Miss Edith Handler for help in editing the manuscript.

[2] *Review of Economic Statistics*, April, 1927, pp. 74–92.

[3] A brief synopsis was given in W. C. Mitchell's *Business Cycles: The Problem and Its Setting*, Addenda, p. 478.

These numbers in one of the original series run from 0 to 9; in two others they are a succession of zeros and ones for the even and odd numbers respectively. The distribution in all these series is obviously rectangular. To the series of numbers from 0 to 9, a ten-item moving average is fitted, and instead of the disjointed, disorderly appearance of the original items there results a line of well-marked cyclical swings. The comparison of a section of this moving average with Dorothy Thomas' Quarterly Index of Business Conditions for Great Britain for the period 1855–77 reveals a striking similarity.

CHART I



It is to be noted that instead of the cumulation (found in the article of Bullock, Persons and Crum) we have here a moving average, *i. e.* a summation complicated by a continuous omission of items one by one. Random data thus yield cycles even when successive items of the generated series are influenced by two items (the item added and the item omitted) and when the number of original items that enter the successive members of the generated series is limited.

In order to make this illustrative experiment more convincing I have tested it, using random distributions of an ascertained nature. Two sources are utilized: an example of a rectangular distribution—the random sampling numbers arranged by L. H. C. Tippett,[1] and an example of an approximately normal frequency distribution, the data on the size of the meal check for a noonday meal in the Commons Cafeteria, Columbia University. The latter data are valuable because the single items have been recorded in the time sequence of their

[1] *Tracts for Computers*, edited by Karl Pearson, No. XV, 1927.

occurrence.[1] The character of their distribution can be seen clearly from the table showing grouping in classes.[2]

In order to confine the data to presentable volume, I had to choose, of course, only sections of the distributions referred to above. From Tippett we have the 50 numbers of column 16, page I, the average for which is 4.3, and the distribution as follows:

Size of item	*Number of items*
0 and 1	11
2 " 3	10
4 " 5	8
6 " 7	15
8 " 9	6
Total	50

The lack of symmetry in the distribution of the larger items would not invalidate the use of the sample, since, if anything, the comparative infrequency of the larger items would militate against the formation of cycles. On the other hand, this particular column is normal in regard to a characteristic to be noted later.

From the data on the meal checks the first 76 items were taken, *i. e.* the first tenth of the total distribution. The average is 37.6 as compared with 38.1 for the total distribution. The grouping is as follows:

Size of item	*Number of items*
.03–.12	1
.13–.22	8
.23–.32	15
.33–.42	25
.43–.52	16
.53–.62	11
.63–.72	0
Total	76

Here a comparative lack of the large positive deviations may again be noted, but the distribution is otherwise normal.

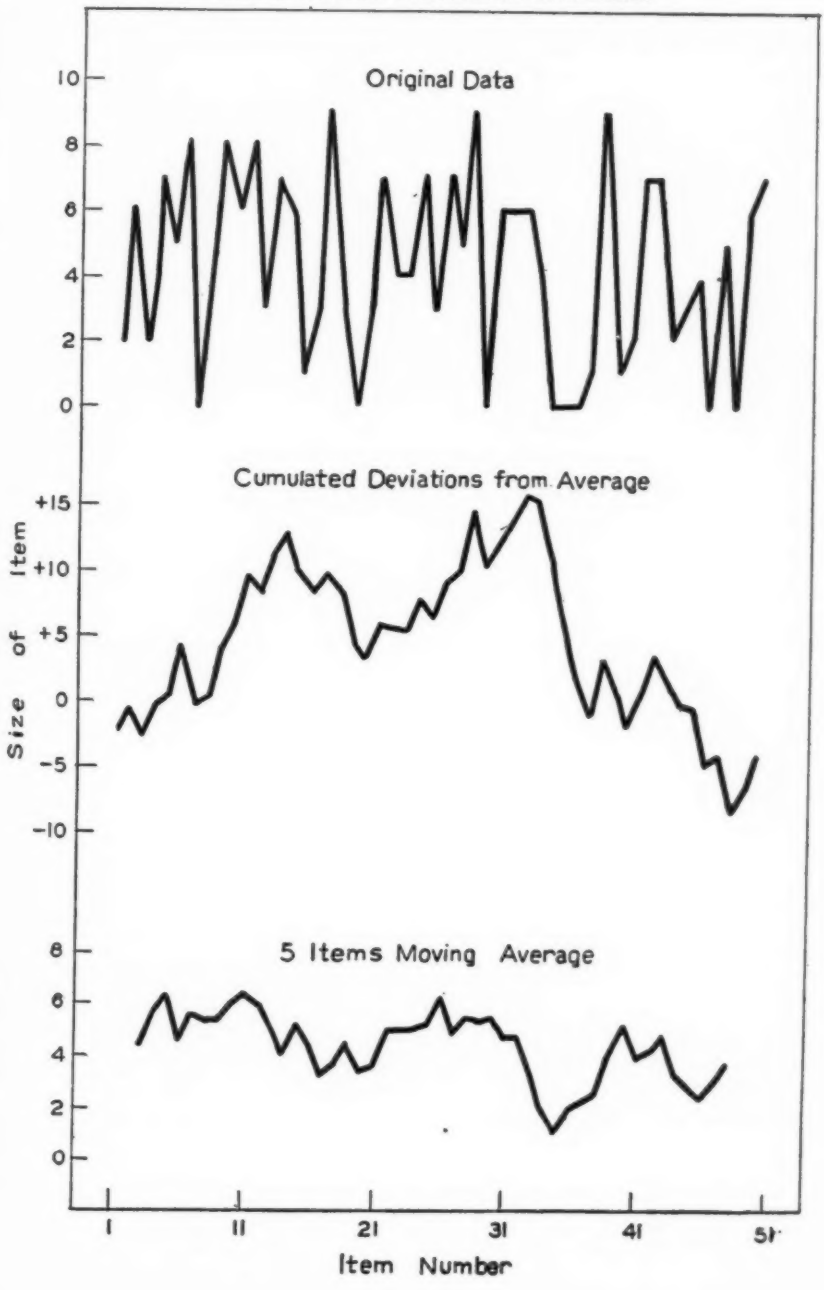
Both of the above samples repeat well the characteristics of their universes, thus justifying their use in our experiments as two fairly regular rectangular and normal frequency distributions.

The results are presented in Charts II and III. In each of these the top line presents the original items in the sequence given in the sources. The middle line presents cumulated absolute deviations from the average. The lower line in Chart II is an arithmetic moving average of 5 items. The lower solid line in Chart III has the same

[1] *Exercises in Statistical Methods* by R. E. Chaddock and F. E. Croxton, p. 63.
[2] *Ibid.*, p. 78.

CHART II

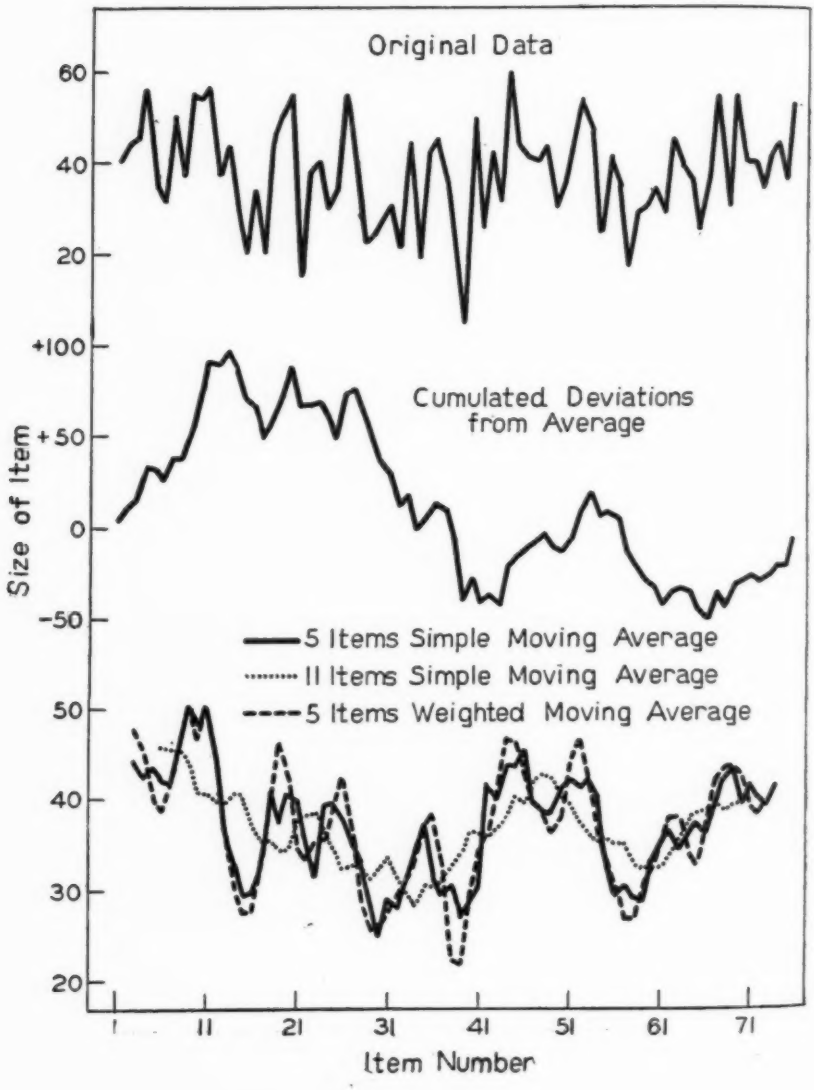
TIPPETT'S RANDOM NUMBERS, ORIGINAL DATA, CUMULATED DEVIATIONS FROM MEAN AND MOVING AVERAGE OF FIVE ITEMS



meaning. The dotted line in the same chart presents a moving average of 11 items computed because the five-item average did not indicate the resulting cycles as clearly as desired. The purpose of the dash line in Chart III will become obvious below.

CHART III

DATA ON THE SIZE OF MEAL CHECKS, NOON MEAL, COLUMBIA UNIVERSITY COMMONS, ORIGINAL DATA, CUMULATED DEVIATIONS FROM THE MEAN, AND MOVING AVERAGES (THREE TYPES)



The comparison shown in the charts indicates that while the original data present scarcely an appearance of the cycle, this latter is quite apparent in the line of cumulated deviations from the mean. The lines of moving averages also indicate rather clearly the cyclical oscillations.

These illustrations are interesting and striking, but the surprising results will become significant only after it is shown why random distributions, when summated, are more or less bound to yield cycles.

II

In explaining the transformation of the disjointed original data into quasi-cyclical oscillations, two elements may be distinguished: First, the change from a sharply disjointed line to a more or less smooth one; second, the appearance of undulatory movements where none previously existed.

The first result is obviously due to the smoothing influence of a process of summation. While the successive items of a chance distribution are entirely uncorrelated, the successive items of a cumulation or of a moving average, having a number of items in common, are closely correlated. Thus in a moving average of 5 items, the resulting members, when taken successively, have 4 items in common and only one item different. The larger the number of items included in the moving average, the closer is the correlation. It is because of this correlation of successive items in the series that we have a gradual change, an appearance of a smooth passage from one point to another, instead of the rugged and porcupine character of the line of original data. In short, we have here the ordinary process of mechanical smoothing.

But why should the resulting smooth line run in waves? Why is it not flat or why does it not run insignificantly up and down around some line of trend?

The reason will be found in

(a) The occurrence of clusters of positive and negative deviations from the average in the distribution of random causes.
(b) The occurrence of large single deviations from the mean.

First, as to the clusters of deviations: If we have a frequency distribution, or any rectangular distribution, there is an equal chance of an item falling either below or above the average (the probability of its being equal to the average itself is so small that it can be disregarded). The probability of an item falling above or below the average is each one-half. The probability of two positive or two negative deviations from the average in succession is, then, one-quarter; of three, one-

eighth; of four, one-sixteenth, etc., according to the theorem of the multiplication of probabilities. Thus if we have a distribution of about 50 items, the only case in which summation of the items would not give us some appearance of the cyclical movements would be a regular succession of positive and negative deviations from the average. In such a case a moving average will yield either a completely flat line or a line repeating the original succession of positive and negative deviations without showing any protracted cycles.

Such a regular succession of positive and negative deviations in the original series is little probable, having only two chances out of the numerous possible combinations; *e.g.* in an original series of four items, the following combinations are equally possible. (Each item is supplied with (+) or (−) to indicate its deviation from the average, an equal number of positive and negative deviations being allowed):

(1) $a(+), b(-), c(+), d(-)$
(2) $a(-), b(+), c(-), d(+)$
(3) $a(+), b(+), c(-), d(-)$
(4) $a(-), b(-), c(+), d(+)$
(5) $a(-), b(+), c(+), d(-)$
(6) $a(+), b(-), c(-), d(+)$

Thus, out of six combinations only two (1, 2) yield a regular succession of positive and negative items, while the other four yield either one or two clusters of two items each. If the number of items is increased, there will still be only two combinations of a regular succession of plus and minus deviations, while all the other combinations will include clusters of varying magnitude.

It is probable, then, that we shall have clusters of positive and negative deviations from the average. It is also probable that when one large cluster occurs it will be followed by clusters of smaller magnitude. Indeed, we know that the average number of deviations running successively with the same sign will be two, or rather will approximate two as the distribution becomes infinitely large.[1] There-

[1] This can be shown when the probable occurrence of each cluster is weighted by its number of items. Thus:

Clusters of	*Probable occurrence* (In terms of total number of clusters)	*Product by number of items*
1 item each	$\frac{1}{2}$	$\frac{1}{2}$
2 items "	$\frac{1}{4}$	$\frac{2}{4}$
3 " "	$\frac{1}{8}$	$\frac{3}{8}$
n	$\frac{1}{2^n}$	$\frac{n}{2^n}$

The sum of the products or the total number of items in the distribution can thus be presented as:

$$\Sigma\left(\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots\ldots\frac{n}{2^n}\right). \quad (1)$$

fore, if there are about 50 items in the distribution there will be about 25 groups or clusters, out of which about one-half will include only one item; one-quarter, two items; one-eighth, three items; and so on. In such a distribution there should be at least one cluster of five items, all either positive or negative.

In turning to our samples to note how the succession of positive and negative deviations run, we find that they correspond to expectation. Indeed in the 50 numbers taken from Tippett we have the following distribution:

ITEMS TAKEN BY DEVIATIONS FROM THE MEAN

Clusters of same sign including	*Number of clusters*
1 item	13
2 items	6
3 "	4
4 "	2
5 "	1
	26

Instead of the expected 25 groups we have here 26, but their distribution is quite regular. It was because of the regularity of this dis-

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This sum can be decomposed as follows:

$$\begin{array}{lll} \text{I} & \dots\dots & \frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ \dots\ +\frac{1}{2^n} \\ \text{II} & \dots\dots & +\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ \dots\ +\frac{1}{2^n} \\ \text{III} & \dots\dots & +\frac{1}{2^3}+\frac{1}{2^4}+\ \dots\ +\frac{1}{2^n} \\ \text{IV} & \dots\dots & +\frac{1}{2^4}+\ \dots\ +\frac{1}{2^n} \\ n & \dots\dots & +\frac{1}{2^n} \end{array} \qquad (2)$$

The expression I is a sum of a geometric progression which tends to approach 1, as n increases indefinitely. Then:

$$\begin{array}{lll} \text{The limit of I} & \dots\dots & 1 \\ \text{" " " II} & \dots\dots & 1-\frac{1}{2^1}=\frac{1}{2} \\ \text{" " " III} & \dots\dots & 1-\left(\frac{1}{2^1}+\frac{1}{2^2}\right)=\frac{1}{4} \\ \text{" " " IV} & \dots\dots & 1-\left(\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}\right)=\frac{1}{8} \\ \text{" " " } n & \dots\dots & \frac{1}{2^n} \end{array} \qquad (3)$$

The sum of expression (1) is thus the sum of 1+ the sum of another geometric progression $\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\ \dots\ \frac{1}{2^n}\right)$, the limit of which is again 1. The total number of items then is $1+1=2$, n terms of the number of clusters. Hence the average size of a cluster is 2 items.

tribution that this particular column was chosen. For here we have the ideal conditions of the chance occurrence of clusters, and the reasons for obtaining cycles through summation become obvious.

The analogous distribution in the data on the size of meal checks both for the 76 items that entered the chart above as well as for the entire 764 cases of the whole array is as follows:

Clusters of deviations with same sign including	*Number of clusters in the sample of 76 items*	*Number of clusters in the total*
1 item	18	176
2 items	8	84
3 "	6	59
4 "	2	22
5 "	2	8
6 "	2	11
7 "	0	2
8 "	0	2
9 "	0	1
10 "	0	1
	37	366

We see that here also the distribution of deviation clusters is by and large in accordance with the expectation based on probability. A succession of a considerable number of either positive or negative deviations is rare, but is bound to occur provided a large enough distribution of the random factors is given.

But if this be the way the positive and negative deviations tend to run in their temporal succession, it becomes obvious why any process of cumulation will tend to give swings very similar to the ones observed in economic time series. If, in the original data there occurs a long succession of high or low items, the cumulative process will summate the positive or the negative deviations from the average and result in a line that rises or declines for a continuous period of time. Thus if we have a cluster of five positive deviations from the mean, the cumulated result will show a rise through all these five points of time, although in the original data there may have been a decline from say the second to the third item. Moreover, if this cluster of five positive items is not succeeded immediately by an equally large cluster of negative items (and this is on the whole little probable), these negative items will merely interrupt the upward movement which will be resumed when the next set of positive items (or when one item) occurs. This upward movement will turn decidedly downward only when a large cluster of negative items occurs or when there is a succession at close intervals of negative items (deviations from the average). The tendency of the

cumulative process is thus to extend the influence of a large cluster of deviations beyond the range of its actual occurrence, until it is counteracted by a similar cluster of the opposite sign.

The same reasoning applies to the effect of the moving average on chance succession of positive and negative deviations from the average. The moving average is but a moving summation, covering a strictly limited period. It can thus smooth out the items only within its own period and can extend the influence of a large cluster of positive or negative deviations only by the approximate length of its own period. If our average happens to be one of five items, each item of the cluster is included in five items of the resulting line, and the influence of the cluster (partly diminished by successive dropping of the members) is thus really significant (beyond its own range) in the neighboring averages that count among its items the members of the cluster. This extends it at most two or three items on either side of the cluster, and not indefinitely forward as in the case of straight cumulation. For this reason straight cumulation yields protracted cycles of considerable amplitude, while the moving average yields shorter cycles of smaller amplitude.[1]

It is clear that the same levelling and extending influence of summation and of the moving average applies to cases in which we have an extremely large deviation from the mean of the distribution. In the original data such a deviation would not tend to form a protracted cycle, unless of course it is preceded and succeeded by items that form a gradual rise or decline. But in a moving average or in a cumulation, this deviation, exceptional in its size, will raise or depress the level of all the members that include it, and instead of one high or low point we get a high or low plateau. The duration of this plateau depends upon the period of the average, or in case of summation upon the sign and size of the following items. Since by nature of the frequency distribution such large deviations will be infrequent, the large positive deviation is unlikely to be followed immediately by an equally large negative one which would cut short its influence. Thus in the extending influence of a summation process coupled with an infrequent occurrence of large deviations there is again a tendency toward formation of cyclical swings, analogous to that resulting from the infrequent occurrence of clusters of deviations of like sign.

It is now clear why the processes of summation yield cycles where in the original data we see only a disorderly succession of positive and negative deviations from the mean. The cycles appear because:

1. The summation binds together the items of the original distribu-

[1] From this point on the exposition departs considerably from Professor Slutsky's memoir.

tion and makes for gradual changes in the resulting generated line.

2. The chance occurrence of clusters of positive and negative deviations and of single large deviations makes the gradual changes appear as prolonged rises and prolonged declines that form the familiar cyclical swing.

III

Hence, since cyclical fluctuations are obtained by summating random distributions spread in time, one becomes interested in the possible variations in this procedure, or in other words, in the influence one or another shape of distribution may have on the cycles, in the significance of the type and period of averaging. In elucidating these, one may only surmise.

The shape of distribution determines the probability of the occurrence of large deviations. In a rectangular distribution, where the large and the small deviations are equally probable, and where the limits of a deviation are rigidly defined, the probability of a large deviation giving rise in the process of summation to an extended high or low plateau is not very high, since for the formation of this plateau it should be followed by a considerable cluster of small deviations. But the expectation of the latter is not any higher than of the large deviation itself, and thus, before long we are likely to get a large deviation of an opposite sign.

In a normal frequency distribution the probable size of the deviation depends obviously upon the shape of the curve. If the curve is peaked, the probability of a large deviation is rather small, that of a small deviation, large. However, when a large deviation occurs it is likely to exert a considerable influence in the process of averaging since it will probably be surrounded by comparatively small deviations. The flatter the top of the frequency distribution the closer does the curve approach a rectangular shape, and the smaller is the likelihood that the large deviation will result in a protracted cycle.

As to the probability of clusters, this is obviously influenced largely by the skewness or lack of skewness of the curve. If there is considerable skewness in one or the other direction, negative deviations or positive deviations become more probable, and thus we may expect larger clusters of one or the other sign. One may assume, although none too surely, that for the purpose of producing cycles a highly skewed curve, with a very narrow peak, is of the best type, having a preponderance of small deviations of a certain sign as well as a considerable number of large ones of the opposite sign. We are thus likely

to have a recurrence of large deviations of one sign followed by clusters of small deviations of the opposite sign.

The influence of the distribution of random factors is, however, less clear and less important than the influence of the type and period of the average used in the smoothing process. We shall omit the straight cumulation altogether, because it is too far removed from reality. In economic life, there is no perpetual influence of an event once occurred on all the subsequent events. Rather, we must conceive of it in the nature of a moving average in the sense that after a certain time its influence is reduced to zero and the item must be omitted altogether from the averaging process.

In the moving average two considerations are of importance: (1) the period taken; (2) the weights assigned to the single values. The period taken determines the time limit within which a certain event is to be counted as an effective cause. It is clear that the longer the period of the average, the longer this extension of an influence of either an exceptional item or an exceptional cluster of items. As a result the cyclical swings will tend to be of longer duration. On the other hand, the exceptional occurrences are combined with a large number of other less exceptional items, and therefore the amplitude is likely to be mild. By the same reasoning a moving average of shorter duration is likely to give a cycle that is shorter on the average and more violent in the amplitude of its fluctuations. For an illustration one might compare the lower solid and dotted lines in Chart III.

The influence of any inequality in weights assigned to the different items in the moving average (considered in their time sequence) on the duration and amplitude of the resulting cycles is equivalent to cutting down the period of the moving average. Giving unequal weights implies that only for some of the items does the influence in its full measure extend over as considerable period of time as it would were the average unweighted. Unequal weights may then be conceived as a combination of unweighted averages of different duration. Their influence on the duration of amplitude of cyclical fluctuations need not be discussed beyond the suggestions made above in connection with the period.

But there is another interesting influence of weighting upon the smoothness of the generated line obtained. Because of the interest of the implications contained in this relationship it might be well to discuss in some detail.

If one examines the lower solid lines in Charts II and III, one cannot help but notice the number of small notches or ripples on the upward and downward swings. They are very similar to those

found in a number of economic series, and are a result of the fact that when a certain rise begins it does not continue uninterruptedly, but is followed at some time or other by a short decline. The rate of change or the first difference taken successively is not uniform in sign, which accounts for the lack of smoothness in the series. Indeed, if we consider how the lines of the moving averages are derived we shall see that while the successive items are closely correlated, the first differences are not. If the period of the average is n then the first difference is $(a_2+a_3+ \ldots\ldots a_{n+1})-(a_1+a_2+ \ldots\ldots a_n)=a_{n+1}-a_1$ and the next first difference is $a_{n+2}-a_2$. It is obvious that since a_{n+1}, a_1, a_2, a_{n+2} are uncorrelated, the successive first differences are uncorrelated.

But what happens when the weights are unequal? We shall consider only one case regarding which we can make a definite statement—the case where the central items are more heavily weighted than the extreme items. In this instance we are bound to have positive correlation among the first differences. Let us consider the simplest possible arrangement of weights for three items of an average. If the weights are 1, 2 and 1 we shall have

1st item of the average	$a+2b+c$	Difference between first and second, $-b+c-a+d$
2nd " " " "	$b+2c+d$	
3rd " " " "	$c+2d+e$	Difference between second and third, $-b+d-c+l$

Here we see that the successive first differences have two members with the same signs, and only one with the sign inverted, while the remaining two are unrelated. Since we have no basis for expecting the one item to be larger than either of the two, we should expect a positive correlation of the successive first differences.

This common element among the first differences may be explained by the fact that from one member of the average to the next there is in both a rise in weights from the first extreme item to the one before the peak and a decline in weights from the one after the peak to the last extreme. The only item that provides for negative correlation is the one which, as c in the example, is just before its peak weight in the first average, at peak in the second, and declines from the peak weight in the third. Since in the other items there are repeated rises and declines in weights, there is positive correlation among the first differences.

That we can usually obtain a smoother line from a moving average whose weights are heavier for the central items can be seen in Chart III. The dash line at the lower end of the chart is a weighted moving

average of five items with its weights distributed along a binomial curve. The weights are successively: 1, 4, 6, 4, 1. When we compare this dash line with a solid line of the simple moving average of five items we see that it escapes a lot of the notches which interrupt the line of this average. Counting the number of changes in sign of the first difference we find that for the simple moving average it was 39 times, while in the line of the weighted average the number of changes was only 17.

But if this be the case, the correlation between the successive first differences of a weighted moving average will tend to be the closer, the larger the number of items in the average. This implies, however, that the other conditions are being kept equal, these other conditions being the size of the rise and decline immediately from and to the peak weight and the character of the distribution of the weights. In order to be sure that the first differences will be positively correlated, the weights must be distributed along a smooth rising and declining line and the difference between the weight just before the peak, the peak weight, and the weight one after the peak, must not be much larger than the other weight differences involved. The significance of these conditions, in view of what has just been suggested as making for positive correlation of first differences is too obvious to need explanation.

These types of weighted averages, *i. e.* averages with weights heavier at the center than at the extremes, with the weights rising and declining along smooth lines, can easily be conceived as a result of repeated averaging with all the weights at one, of repeated simple averaging. Indeed, if we take an unweighted average for any period, and repeat the averaging successively we obtain an average whose weights are heaviest at the center, whose weights are distributed symmetrically around the center, rising and falling uninterruptedly (although not in a smooth line, unless the number of repetitions is large), and an average where in the weight distribution the weight difference immediately preceding and succeeding the peak weight is either equal to or smaller than one of the weight differences among the most extreme items. (A specific case of such a weight distribution is the binomial weight distribution which results from a successive repetition of simple averaging by two.)

The symmetrical character of such a weight distribution is obvious, since it is a result of the equality of the original weights and of the successive inclusion of a larger and larger number of items in the process of successive averaging. The size of the weight difference is determined by the fact that each weight in a given averaging is equal

to the weight of the same item in the preceding averaging plus the weight of the preceding item in this same earlier averaging. Without going into the mathematics of the case, we can show that for our purposes, for the purposes of proving the probability of positive correlation among the first differences, the relatively small size of the weight difference at peak holds true.

Thus we know that if the number of items in the average is even, the peak weight is going to be given to two items. When, therefore, we take the successive first differences, we do not have even the one element of negative correlation mentioned above, since we have only a contrast of a positive or negative sign with zero, and not with its reverse sign. Here obviously the successive first differences are certain to be positively correlated. In the next averaging case, where we have an odd number of items in the average, the second of these earlier two peak weights is going to become the only peak weight. But the weight difference between it and the preceding weight will be equal only to the difference between the weights of its two predecessors in the earlier averaging, while the weight difference of the two more extreme items will be compounded of two sets of weight differences, one of which is the same that accounted for the formation of the peak weight. The weight difference around this new peak will be necessarily smaller by the fact that the peak has arisen from weights, equal in the preceding averaging, while the other weight differences are compounded of the difference that has existed in the preceding averaging plus the difference added by the new averaging, the latter being the one which now forms the weight difference around the peak. Thus when the number of items is odd, the weight difference around the peak is smaller than the weight difference one item further away from the center.[1]

[1] We might make this more obvious by a simple example. If a simple average of three items is taken, the first repeated averaging yields the case of two peak weights, as follows:

$$\begin{array}{r} a+b+c \\ +b+c+d \\ \hline a+2b+2c+d \end{array} \qquad (1)$$

The first difference is then:

$$(b+2c+2d+e)-(a+2b+2c+d)=-b-a+d+e. \qquad (2)$$

The next first difference is:

$$(c+2d+2e+f)-(b+2c+2d+e)=-b-c+e+f. \qquad (3)$$

We see that in the two differences (2) and (3) there are no negatively correlated items, and two (b and e) positively correlated.

In the next averaging c and d respectively will become the peaks. Thus:

$$\begin{array}{rr} a+2b+2c+d & b+2c+2d+e \\ b+2c+2d+e & c+2d+2e+f \\ \hline a+3b+4c+3d+e & b+3c+4d+3e+f \end{array} \qquad (4)$$

One can see that the difference between the peak weight and the preceding weight is compounded of $(2-2+2-1)=1$, while the difference at the extreme is $(2-1+1-0)=2$.

The first difference is:

$$(b+3c+4d+3e+f)-(a+3b+4c+3d+e)=-a-2b-c+d+2e+f.$$

Thus the weight distributions that result from repeated simple averagings tend to make for close correlation among the first differences, and therefore for a smooth curve of cyclical behavior.

To summarize the results of this discussion:

1. Following Professor Slutsky's exposition, it was indicated that distributions of random causes, when summated, yield oscillations similar to those characteristic of economic data.

2. It was shown that this result is due to the smoothing and carrying over power of the summation process, and to the fact that in a frequency distribution of random character we are bound to find big clusters of deviations above or below the average, or single exceptional deviations. Of these, the process of summation or of moving averages tends to make cycles.

3. The shape of the distribution of the random causes and the period of the moving average will influence the amplitude and timing of cycles thus resulting. A peaked distribution skewed considerably to one side was suggested as the most likely source of clear-cut cycles. A moving average of a large number of items tends to produce long and mild cycles, a shorter average—shorter and more fluctuating cycles.

4. It was shown that a moving average with its central items weighted more heavily than the extreme ones, tends to give a much smoother line than a simple moving average, because it makes for the correlation of successive first differences. And it was shown that a weighted moving average, with its weights distributed symmetrically along lines of uninterrupted rise and decline, the rise and decline slowing down around the peak, can be conceived as a result of repeated averagings of a simple type.

IV

In discussing the implications of these conclusions, one must establish first of all the legitimacy of what might be called the inverted inference. It has been shown that the summation of random causes yields cycles, that certain peculiarities of the distribution of these random causes and of the averaging process influence the characteristics of the cyclical oscillations obtained. But can one invert the proposition and say that, therefore, cyclical oscillations may be conceived primarily as results of summation of random causes, and that the characteristics of some

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Similarly the next first difference is: $-b-2c-d+e+2f+g$.

Only the d is negatively correlated; while there are four items with the same sign. Thus also in this case the successive first differences are positively correlated.

It is obvious that the two cases discussed here are typical of all the even and odd items of weighted moving averages.

of these cyclical oscillations can best be grasped as a result of some peculiarities of the underlying random events or of the process of cumulation?

Such inference, of course, cannot carry with it any certainty, since we are never certain of all the contingencies of other hypotheses to which the formations of cycles in economic data may be reduced. But the only way to establish the significance of the conclusions stated, is to proceed from it as from a significant but not exclusive hypothesis and to see how it agrees or disagrees with the other elements of the known universe which are associated with the cyclical oscillations. This is exactly what we are doing when we trace the implications.

One statistical implication is obvious. We are debarred from treating time series as congeries of units distributed around the average (the trend line) as if they were a frequency series. The conception of them as a moving average obviously predicates a close correlation and not independence of successive items. The closer the correlation, the smoother the line of cyclical movements looks.

The other statistical implications are not clear at the present moment. One might attempt to study the periodicity of cycles as a problem of average periodicity of occurrence of clusters of deviations of same sign or of an especially large deviation. One might think of decomposing each time series into the probable distribution of random events and the probable processes of summation whose combination would give us the analyzed time series. One might go ahead with Professor Slutsky in an attempt to present time series as a result of smoothing chance distributions by a binomially weighted moving average and attempt to derive from it the characteristics of their behavior and of their correlation. However, the economic conditions that underlie time series suppose that a chance distribution of events occurs only within narrow limits and does not permit us to treat the rises and declines in a time series as continuous cumulations of economic events that are continuously random in their character.

But it is the implications for the theory of economic cycles which are most obvious and most interesting in this connection. I shall confine myself only to a brief enumeration.

1. If cycles arise from random events, assuming the summation of the latter, then we obviously do not need the hypothesis of an independent regularly recurring cause which is deemed necessary by some theorists of business cycles. Indeed, if one can explain how in certain processes of economic life, the response to stimuli is cumulative, then the whole discussion of the cause of business cycles becomes supererogation. If the business economy runs at a certain high or low plateau

level, and the conditions of business behavior are in such shape as to cumulate favorable or unfavorable random events, then we are bound to have a cyclical up or down swing sooner or later. It is to be seen that the so-called institutional explanations of business cycles deal mainly with the economic forces that make for cumulation, with forces that explain why a given random event is not immediately cancelled by an opposite reaction but allowed to exert its influence for some time to come, an economic counterpart of the statistical mechanism of a moving average.

2. If we observe smoothly running cyclical oscillations, as we usually do in such indices as the index of wholesale prices, or the index of production, this smoothness may be conceived as a result of repeated summation of the random events that underlie the cycles in each of the single, specific indices entering these general indices. In other words, in this repeated averaging, in one effect becoming in its own turn a source of another effect, we have the explanation of the smooth behavior of this series. It implies the heavier distribution of weights at center as discussed before and also the extension of the period of influence. If, on the other hand, we have a choppy, highly fluctuating series, like the number of shares sold on the New York Stock Exchange, we may ascribe it to the short period during which the chance events are allowed to operate, and the corollary that no repeated conversion of cause into effect and of effect into a wider cause is possible.

These are some of the connections that suggest themselves. It seems to me that in the conception of the oscillating time series as a summation of random causes lies one of the first significant links between the general theory of frequency distribution and time series. It also throws a great deal of light on some of the purely mechanical characteristics of the cyclical oscillations in economic time series.

SOME OBSERVATIONS CONCERNING THE ASSUMPTION OF NORMAL WORKING TIME

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Much of the reasoning concerning wages is based upon normal working time. Just what is meant by this expression is not always clear. In some instances it implies uniformity in nominal time, as when all establishments have a full-time working period of 48 hours per week. In other instances it implies that actual and nominal working time are identical. Frequently these two related, although distinct, meanings are not differentiated.

When normal time is assumed merely to estimate *potential* rather than *actual* earnings, or potential rather than actual hourly rates, then the accuracy of the assumption may not be particularly important. But when the assumption is employed in the calculation of actual wages or in making comparisons of actual wages its validity becomes quite significant. For example, it is frequently necessary to have information as to both rates and earnings. If only rates are known, weekly earnings may be calculated on the basis of an assumption as to actual working time. Or if weekly earnings are known, hourly earned rates may be calculated on the same basis. This same procedure is followed at times in the collection of wage data. Instead of collecting both rates and earnings, the rates are likely to be obtained and the earnings calculated from them. Such calculations are usually made on the assumption of normal employment or normal working time, and in such instances it becomes necessary to know not only whether actual and normal time do correspond, but also whether normal working time is uniform for all, or substantially all, men and plants involved. If normal or nominal time is not a uniform period, say 48 hours, in all instances, then what relation is there between the differences in normal time and both rates and earnings? Both of the aforementioned implications of the expression "normal working time" are, therefore, involved when rates are calculated from earnings and earnings from rates.

Recently there has been an opportunity to examine some aspects of this assumption in the light of minute information supplied in a particular industrial center of the United States. With the coöperation of the Metal Manufacturers' Association of Philadelphia, 83 metal manufacturing plants furnished information concerning actual earnings and

time actually worked for a week during the month of April, 1927. This and certain other information for every employee was supplied to the Industrial Research Department of the University of Pennsylvania.[1] In order that more extensive information might be obtained on certain points, 12 plants coöperated in supplying information which included time actually worked, week by week, for the men in 5 occupations for the years 1925 and 1926.[2]

For convenience, attention will here be confined to only 1,456 of the 25,459 employees reported in April, 1927. The 1,456 employees are all men and they are distributed among 7 standard machine tool occupations in 43 plants. The occupations are: drill press operating, milling machine operating, turret lathe operating, screw machine operating, engine lathe operating, planer operating, and boring mill operating.[3] The 1925 and 1926 data covered 652 and 1,086 men respectively, distributed among 5 [4] of the 7 occupations named above.

The subsequent analysis [5] centers around three points: First, what is meant by normal working time in connection with these occupations; second, what bearing does this meaning have on the calculation of rates from earnings and earnings from rates; third, would the assumption have been valid in this case that actual time corresponds with normal time?

To notice that the normal or nominal number of hours per week worked in machine shops in the United States varies from 44 to about 55 may not be particularly impressive in general comparisons. One is prone to think that this spread of about 11 hours reflects differences in the industrial conditions in various parts of the country, and that in important centers normal time is likely to be much the same. But when one finds substantially the same limits, and hence the same spread, of normal hours in a few occupations among plants in a given city, then the variations in normal time take on greater importance so far as the meaning of the term "normal working time" is concerned.

[1] This survey was under the direct supervision of Miss Anne Bezanson, Associate Director of the Industrial Research Department, assisted by Miss Miriam Hussey of the same department. The participation of the author was confined to those occupations in which he was especially interested.

[2] This survey, directed by the author as a member of the Industrial Research Department, was begun in the latter part of 1927, and at that time the records for the years 1925 and 1926 were the most recent ones which could be used (and in some cases taken from the plant) without interfering with the current operation of the plant.

[3] These occupations are among the comparatively few that are very important throughout modern industry where speed, accuracy, and power are so essential.

[4] Planer operating and boring mill operating not included.

[5] It is impossible for the author to acknowledge specifically his indebtedness to all those persons who were of assistance to him. Particular credit is due to members of the Industrial Research Department, especially to Dr. Joseph H. Willits and Dr. Anne Bezanson, Director and Associate Director, respectively, and to Miss Miriam Hussey. Dr. Paul Gemmill of the Economics Department made helpful suggestions and the coöperation of Mr. Earl Sparks, secretary of the Metal Manufacturers' Association of Philadelphia, was very valuable.

From Table I it may be seen not only that there is a spread between 44 and 55 hours a week, but that the bulk of the men and of the plants have normal time at or between 48 and 55 hours. In fact, there is a considerable concentration of both men and plants at two periods. There are 40 per cent of the men and 23 per cent of the plants with normal time of "48 and under 49 hours," while 25 per cent of the men and 19 per cent of the plants have a normal week of "55 and under 56 hours." Thus between the points at which there is the greatest concentration there is a difference of about 7 hours per week.

TABLE I

NUMBER AND PERCENTAGE OF MEN AND OF PLANTS WITH SPECIFIED PERIODS OF NORMAL TIME PER WEEK

Normal Hours	Men		Plants	
	Number	Percentage	Number	Percentage
44 and under 45	11	1	1	2
45 and under 46	56	4	3*	7
46 and under 47	..	..	..	..
47 and under 48	..	..	..	..
48 and under 49	585	40	10	23
49 and under 50	47	3	6	14
50 and under 51	69	5	2	5
51 and under 52	76	5	3	7
52 and under 53	100	7	4	9
53 and under 54	61	4	1	2
54 and under 55	86	6	5	12
55 and under 56	365	25	8	19
Total	1,456	100	43	100

* One of these plants has a few men on a 50-hour basis.

This means that for certain purposes and under certain circumstances normal working time must be defined as a series of periods occurring within a *range* of hours per week rather than as a *uniform number* of hours. Such a definition is, of course, cumbersome, but this does not justify a simplified assumption which not only may be contrary to fact but which may conceal important underlying considerations. Nor can it be argued that such variations in normal time are not equally characteristic of all industries. This is beside the point. The fact that the metal industry is an important one, and that an analysis in one of the principal industrial centers of the world shows marked variations in normal time, prevents their casual disregard.

The question now arises as to what bearing these variations in normal time have on rates and earnings, and hence on the calculation of the one from the other.

Assume two plants to have the same base rate of 60 cents per hour

for approximately the same work. The one plant operates 48 hours per week and the other operates 52 hours. The weekly earnings of a full-time man in the first plant are $28.80, while the man in the second plant receives $31.20. Over a period of 50 weeks this difference amounts to $120, which means, other things being equal in the two plants, that the man in the 48-hour plant loses the earnings of about 4 weeks in a year. Although he does not work these 4 weeks, it is highly questionable whether a man with weekly earnings in the neighborhood of $30 will have such a high regard for leisure that he is willing to sacrifice the income of more than a month's work to have such leisure.

In this illustration a difference of only 4 hours per week has been used. Actually the difference in normal working time in the market amounts to 11 hours, or nearly 3 times that assumed above.

On the other hand, if the worker in the 52-hour plant received higher hourly earnings than the worker in the 48-hour plant, then the difference in weekly and yearly earnings would be even greater. Assuming that the hourly rate is increased by one-half for each hour over 48, so that the man working 52 hours a week has a rate of 62 cents, then there would be a difference of about $292 a year. This means that the man in the 48-hour plant loses the equivalent of about six weeks possible working time. When the difference in normal time exceeds the 4 hours here assumed, the difference in earnings is even greater.

A third alternative is that the plant with the longer normal week may pay less per hour than the plant with the shorter week. In this case how will the weekly earnings compare for the plants? Is it necessary for plants with the longer week to pay an hourly rate which will yield as much as, or must it yield more than, a higher rate yields in a plant with the shorter week?

In order to analyze the relation of variations in normal working time to rates and earnings, the different periods of normal time will be separated into two groups: under 50 hours and 50 hours and over per week. By using these periods both the number of men and the num-

TABLE II

	Men		Plants		Man Hours	
	Number	Percentage	Number	Percentage	Actual	Normal
Under 50 hours	724	48	20	47	33,280	33,181
50 hours and over	732	52	23	53	39,980	40,815
Total	1,456	100	43	100	73,260	73,996

ber of plants are about the same in each period. Moreover, in a given period the total actual and total normal man hours correspond quite closely.

From Table III it appears that the average earned rates or average hourly earnings tend to vary inversely with the duration of working time per week. In this table the data have been separated not only on the basis of working time and occupations but also on the basis of method of payment. The upper part of the table gives the rates for all the men in each occupation (a total of 1,456), and the lower part gives the same information for only those men in each occupation who actually worked the normal time of their plants (a total of 760).

TABLE III

SUMMARY TABLE OF AVERAGE EARNED RATES BY OCCUPATIONS AND BY TIME, PIECE, AND BONUS METHODS OF PAYMENT IN EACH OF TWO PERIODS OF NORMAL WORKING TIME

(In cents)

SECTION A—TOTAL CASES, 1,456

Occupation	Method of Payment					
	Time Basis		Piece Basis		Bonus Basis	
	Under 50 hours per week	50 hours per week and over	Under 50 hours per week	50 hours per week and over	Under 50 hours per week	50 hours per week and over
Drill press operating	61.7	55.4	66.3	56.7	73.2	64.0
Milling machine operating	64.2	63.4	67.4	68.7	77.7	87.1
Planer operating	68.3	65.9	71.9	70.9	84.2	73.4
Turret lathe operating	70.5	64.4	70.7	71.0	84.3	68.2
Screw machine operating	65.9	58.3	72.7	53.8	76.2	80.1
Engine lathe operating	68.4	64.6	72.5	76.5	88.2	74.2
Boring mill operating	75.4	69.5	80.0	70.9	96.5	77.8
Combined occupations	66.2	63.5	69.7	71.6	84.2	75.3

SECTION B—TOTAL CASES, 760

Drill press operating	59.5	52.5	64.6	52.2	74.0	69.0
Milling machine operating	63.8	63.5	69.1	62.5	77.5	82.3
Planer operating	60.0	63.8	72.0	63.8	79.0	74.0
Turret lathe operating	70.6	65.0	68.2	67.0	90.4	67.9
Screw machine operating	66.9	54.6	71.6	58.7	76.0	73.7
Engine lathe operating	67.0	64.7	73.1	77.2	83.2	77.6
Boring mill operating	72.5	69.6	80.3	73.0	99.0	73.0
Combined occupations	65.0	62.5	70.0	68.1	82.8	74.5

Throughout both parts of Table III there is a tendency for the rates to be higher in the shorter than in the longer period of time. This tendency is especially strong where men are paid on a time basis.[1]

[1] The only exception occurs in the occupation of planer operating, where there is a total of only 58 men.

And it is with a time method of payment that the calculation of rates from earnings and of earnings from rates is particularly significant, for here the earned rate may be compared directly with nominal rates. Incentive methods of payment present some technical difficulties which are avoided when men are paid on a time basis.[1]

Most of the exceptions to the tendency for rates to vary inversely with the duration of the normal week occur in connection with piece work. Here it happens that there are three plants with relatively large numbers of piece workers, and the normal time of two of these plants falls into one period.

The relation of weekly earnings to the duration of normal time is not so clear. There is some indication that earnings tend to be higher in the plants with the longer normal time per week than in the plants with the shorter normal time. But there are numerous exceptions, and too definite a conclusion is not warranted.

Table IV gives the average weekly earnings for time, piece, and bonus methods of payment for each occupation in the two periods of normal time. Sections A and B of the table give the average actual and average potential [2] earnings, respectively, for all the men in each occupation, while Section C gives the average actual earnings for only those men who really worked the equivalent of the normal time of their plants.

There is a more marked tendency for earnings to be higher in the longer than in the shorter period for time workers than for men paid by other methods. Excluding potential earnings, the most numerous exceptions to this tendency occur with piece workers, and this is substantially the same situation as occurred when rates were being considered.

We conclude that normal time among operators of standard machine tools in the Philadelphia area is not a uniform period but rather a series of periods distributed within a range. This fact has an important relation to rates and earnings, since rates show a strong tendency to vary inversely while earnings show a less marked tendency to vary directly with the duration of working time per week. That these relationships affect, adversely, the calculations of rates from earnings and earnings from rates seems to be frequently overlooked or disregarded. This statement is particularly applicable when the calcula-

[1] Three different methods of payment are recognized here—time, piece, and bonus—because there is a persistent tendency for both earned rates and weekly earnings of piece workers to exceed those of time workers and to fall below those of bonus workers. A fuller treatment of this will be given in *Earnings in Certain Standard Machine Tool Occupations in Philadelphia*, by the present author, and published through the Industrial Research Department by the University of Pennsylvania Press.

[2] Average potential earnings are those earnings which would have accrued if men had worked the normal time of their plants at their actual earned rates.

TABLE IV

SUMMARY TABLE OF AVERAGE WEEKLY EARNINGS BY OCCUPATIONS AND BY TIME, PIECE, AND BONUS METHODS OF PAYMENT IN EACH OF TWO PERIODS OF NORMAL WORKING TIME

AVERAGE ACTUAL EARNINGS, SECTION A—TOTAL CASES, 1,456

Occupation	Method of Payment					
	Time Basis		Piece Basis		Bonus Basis	
	Under 50 hours per week	50 hours per week and over	Under 50 hours per week	50 hours per week and over	Under 50 hours per week	50 hours per week and over
Drill press operating	27.67	30.13	34.85	30.52	31.62	32.19
Milling machine operating	29.77	32.73	33.98	34.92	35.47	46.13
Planer operating	40.76	37.77	34.52	37.69	34.36	37.69
Turret lathe operating	29.45	35.31	32.18	37.86	35.56	33.96
Screw machine operating	31.51	31.12	35.41	26.74	37.72	39.70
Engine lathe operating	31.09	34.56	34.75	40.46	37.47	38.10
Boring mill operating	31.53	36.91	38.08	37.12	41.51	41.34
Combined occupations	30.53	34.15	34.87	37.78	36.14	38.33

AVERAGE POTENTIAL EARNINGS, SECTION B—TOTAL CASES, 1,456

Drill press operating	29.06	29.58	31.82	29.88	35.28	35.07
Milling machine operating	30.50	33.48	32.02	36.07	37.45	47.21
Planer operating	30.87	35.98	34.51	38.36	40.42	39.86
Turret lathe operating	33.84	34.26	32.45	37.84	40.72	35.46
Screw machine operating	31.37	30.20	34.90	28.30	37.72	42.45
Engine lathe operating	32.76	33.85	34.22	41.46	42.51	39.85
Boring mill operating	35.74	37.60	38.24	38.64	46.51	42.40
Combined occupations	31.45	33.59	33.32	38.59	40.58	40.36

AVERAGE ACTUAL EARNINGS, SECTION C—TOTAL CASES, 760

Drill press operating	27.96	27.88	31.05	27.30	35.73	37.86
Milling machine operating	30.53	33.45	33.15	31.83	37.30	44.72
Planer operating	27.45	34.82	34.53	34.49	37.97	39.93
Turret lathe operating	33.92	35.12	31.13	35.29	44.11	35.16
Screw machine operating	31.79	28.35	34.39	31.09	37.72	39.27
Engine lathe operating	32.21	33.92	34.65	41.43	39.97	41.41
Boring mill operating	34.80	37.66	38.08	38.82	47.56	39.68
Combined occupations	30.96	33.11	33.53	36.04	39.95	39.83

tions are designed to show actual wages rather than changes in wage levels, but it may be equally important when the purpose of the calculations is to compare the general level of wages in an occupation, or for a grade of workers, at one time with the general level at another, provided there have been marked changes in normal working time.

Turning now to the second sense in which the expression "normal working time" or "normal employment" is often used, *i. e.*, implying that actual and normal time correspond, the analysis will be twofold. First, the pragmatic test of actual aggregate results will be used. Judged solely on this basis, is there evidence that actual time did correspond with normal time? This conclusion will later be examined

in the light of the underlying situations, which are important not only where rates are calculated from earnings and earnings from rates but also where wage data are compiled, or actually used, in connection with wage policies of individual plants or of an entire industry.

Let us assume first that the earned rates and the normal time of the workers are known. How, then, do the resulting average potential earnings compare with the average actual earnings? The answer is that the results are much the same in each occupation. This may be seen by comparing the figures in columns A and B of Table V.

Now suppose that the normal time of the workers is not known and that an assumption of 50 hours per week is made. On the basis of known earned rates, how do the average estimated and the average actual earnings compare? Again there is a surprising similarity in the averages, as shown in columns B and C of the table.

TABLE V
SUMMARY TABLE OF AVERAGE WEEKLY EARNINGS, AVERAGE HOURLY EARNINGS, AND TOTAL MAN HOURS IN SPECIFIED OCCUPATIONS

Occupations	A	B	C	D	E	F	G	H
Drill press operating	31.66	32.29	31.85	30.36	63.7	61.0	12,182	12,446
Milling machine operating	34.40	34.93	34.75	34.49	69.5	69.1	10,238	10,385
Turret lathe operating	36.96	35.52	35.95	36.23	71.9	70.9	6,428	6,230
Screw machine operating	35.37	35.20	35.95	34.16	71.9	69.8	12,785	12,735
Engine lathe operating	38.25	37.02	36.85	37.09	73.7	72.6	21,390	20,809
Planer operating	37.47	37.25	35.75	35.85	71.5	68.5	3,040	3,044
Boring mill operating	39.37	37.68	37.00	39.05	74.0	75.7	7,933	7,612
Combined occupations	36.02	35.55	35.45	34.89	70.9	69.4	73,996	73,261

A—Average potential weekly earnings (1,456 men).
B—Average actual weekly earnings (1,456 men).
C—Average estimated weekly earnings (1,456 men).
D—Average actual weekly earnings (760 men).
E—Average earned rates (1,456 men).
F—Average earned rate (760 men).
G—Total normal man hours (1,456 men).
H—Total actual man hours (1,456 men).

A third possibility is the relation of potential and actual earned rates. Such a comparison requires a calculation based upon the actual earnings and the normal time. This calculation is not readily available, and since short time and over time do not influence earned rates nearly so much as weekly earnings, especially when the rates are calculated on the basis of weekly data, it has been deemed advisable to omit this comparison.

Thus far all workers in the various occupations have been included in the comparisons. In the next two comparisons it will be assumed that certain data are available for only those men who actually worked the equivalent of the normal time of their plants. Then the question will be: How do these data compare with similar data for all the men in the several occupations?

Assume first that the actual earnings are known for the men who worked normal time. How do these earnings compare with the earnings of all the men in the respective occupations? Once more there results a striking similarity, as is seen by comparing the figures in columns B and D of Table V.

Instead of knowing the actual earnings, suppose the earned rates had been known; would these also be similar to the rates for all the men in the respective occupations? The answer is yes, and the basis for it is shown by the figures in columns E and F.

Thus by a series of tests it appears that in the aggregate the results are very similar to results which either did occur or might have occurred with normal working time. In short, there is some basis for the inference that actual working time corresponds rather closely with normal time, and it would seem, therefore, that over time and short time were relatively unimportant in this period.

Judged on the basis of man hours, there is a very marked similarity between total actual and total normal man hours. Columns G and H show the former to be 73,261 and the latter 73,996 for the combined occupations. Similar likenesses occur with the individual occupations.

It is now in order to inquire whether actual and normal time did in reality correspond as closely as the foregoing similarities might suggest. Would it have been valid to assume that over time and short time were of no significance in this period?

This question may be approached from the standpoint either of the market in general or of the individual plants. For the market in general, the April, 1927, data show that 52 per cent of the 1,456 men worked the equivalent of the normal time of their plants in a period of one week, while 28 per cent worked less and 20 per cent worked more than this time. Thus any assumption as to actual time corresponding with normal time would be applicable to only about 50 per cent of the men.

Aside from the rather appreciable proportion of men who did not work normal time, the second most striking feature is the percentage of men who worked over time. This is particularly important in view of the condition of business in April of that year. As judged from the average number of men on the payrolls of 52 metal manufacturing plants in Philadelphia, business activity was on the decline. Beginning in February, the average number began to decline and it continued throughout the year, with a much greater decline between March and April than between February and March.

Before the degree of short time has much significance, it is necessary to make allowance for entrances and separations during the week.

TABLE VI

AVERAGE NUMBER OF MEN ON PAYROLLS OF 52 METAL MANUFACTURING ESTABLISHMENTS IN PHILADELPHIA BY MONTHS FROM JANUARY, 1927, TO JANUARY, 1928 *

	Average number on roll
1927	
January	35,946
February	36,589
March	36,078
April	34,778
May	34,960
June	33,222
July	32,579†
August	32,565
September	32,209
October	30,875‡
November	30,388
December	30,780
1928	
January	31,463

* Data compiled by Metal Manufacturers' Association of Philadelphia.

† One company, with 268 men in June, omitted in July and subsequent months.

‡ Two companies, with 56 and 169 men in September, omitted in October and subsequent months.

Though this has not been done, it in no way affects the previous observation that over time seems to have been rather extensive for a period of declining business activity. And this fact causes one to wonder how extensive over time might be in a period of increasing activity.

Judged by yearly data, even fewer men worked the normal time of their plants. So far as the present argument is concerned, only April, 1927, data are directly applicable. At the same time it is relevant to know how actual working time compared with normal time when longer periods were examined. Out of 652 men reported in 1925, and 1,086 in 1926, only about 40 per cent each year worked 49 weeks or more in the plant from which they were reported. Average actual working time per week was computed for these men, and only 9 per cent of them had an average number of hours per week equivalent to the normal time of their plants.

Those men who did not work the equivalent of the normal time of their plants were widely distributed among the plants.[1] In 1927 there were only 11 out of a total of 43 plants in which all the men worked normal time and there were only 296 out of a total of 1,456 men in these 11 plants. In 1926 and 1925, where the observations covered the respective calendar years, there was not a single plant where the men

[1] For a fuller treatment of this point see *Earnings in Certain Standard Machine Tool Occupations in Philadelphia.*

who worked 49 weeks or more in a year had an average number of hours per week equivalent to the normal time of the plant. And the situation is not materially changed when the average actual time of a plant is compared with its normal time. In 1927 there were 13 plants, in 1926 there was one plant, and there were none in 1925 where the average actual time of the plant corresponded with its normal time.

While the non-normal time cases are widely distributed among the plants the degree to which actual and normal time correspond varies considerably between the plants. If the actual time is expressed as a percentage of normal time it appears that in 1927 the percentages range from 71.4 to 123.1; in 1926 from 89.3 to 110.7; and in 1925 from 87.1 to 106.9.

There is no intent on the part of the writer to allocate responsibility for any of the situations here described. The only point of interest is the relation of actual to normal time. And the conclusion to which this analysis points is: (1) that in a period of a week a larger proportion of men actually worked the normal time of their plants than in a period of a year; but (2) even in a period of a week the degree to which actual time corresponds with normal time is not that which might be inferred from the earlier hypothetical situations and from the marked similarity between total actual and total normal man-hours.

Since only about 50 per cent of the men worked normal time and since average earnings for all men in an occupation are approximately the same as the earnings for those who worked normal time, the question may be raised as to whether there is actually much difference between the average earnings of men who worked short time and those who worked over time. The answer is that there is considerable difference. As shown in Table VII, the average earnings for men working less than normal time are at least several dollars below the average for the men who did work normal time in all but one occupation, where an unusual situation occurs. A much greater difference, however, is found between the average earnings for men who worked normal time and those who worked more than normal time. The influence of over time made a considerable difference in average earnings, and this difference is not entirely due to a few exceptional cases. True, there are such cases, just as there are some exceptionally low earnings among the men who worked short time; but the *general level* of earnings for men who worked over time is appreciably higher than the level for the men who worked only normal time.

In short, to assume that the influence of over time and short time are negligible because the average earnings for the men who worked normal time correspond with the average earnings of all the men in an occupa-

TABLE VII

AVERAGE WEEKLY EARNINGS FOR MEN WHO WORKED UNDER NORMAL, NORMAL AND OVER NORMAL TIME IN SPECIFIED OCCUPATIONS

Occupations	Under normal time	Normal time	Over normal time	Total
Drill press operating	27.08	30.36	40.04	32.29
Milling machine operating	31.28	34.49	40.09	34.93
Turret lathe operating	27.91	36.23	46.47	35.52
Screw machine operating	35.70	34.16	40.96	35.20
Engine lathe operating	32.19	37.09	44.14	37.02
Planer operating	33.77	35.85	52.23	37.25
Boring mill operating	31.41	39.12	47.85	37.68

tion, or because total actual and total man hours correspond, is not warranted in this instance. The same similarity in earnings might arise from a balancing of the influence of over time against that of short time, so that the average earnings for all men in an occupation would be the same as though there had been neither over time nor short time. Such a balancing is perhaps unusual, but it appears to be the case here. And certain very pressing problems may be concealed under an apparently satisfactory general situation.

The foregoing analysis suggests that in connection with wage discussions the expression "normal working time" may require more careful definition than it often receives when the expression is used to denote a fact. For many purposes it is significant whether this term refers to a uniform number of hours or to a series of hours.

Moreover, when the expression is used as an assumption that actual and normal time correspond, it may require more careful scrutiny than it usually receives, even when it plays a vital part in an argument. While surface indications suggest, and the aggregate results confirm, marked similarity of actual and normal time, it may still be incorrect to infer that this similarity is representative of the particular situations which give rise to the aggregate conditions.

THE MEASUREMENT OF THE DEMAND FOR FOOD

By Faith M. Williams

Recent studies of the economic aspects of food consumption by farm, village and city families emphasize the necessity for revising the units commonly used in measuring the demand for food. The very nature of the problem makes the formation of such units extremely difficult, but the rapid development of the science of nutrition in the last quarter century makes it possible to improve the measures now in use. A history of the development of these measures will clarify the problem.

The earliest students of food consumption were content to base their calculations on the amount of food consumed, and food expenditure *per family*. The difficulties involved in thus comparing family with family without further analysis were soon recognized, however, and the careful students of consumption devised various methods of remedying the situation. The International Statistical Congress of 1853 recommended using figures from "typical" families only. In 1875, Carrol D. Wright, then Chief of the Massachusetts Bureau of Statistics of Labor, published the results of a survey of the earnings and expenditures of 397 Massachusetts wage-earning families. The general summaries of the data gathered show that the average family surveyed consisted of 5.14 persons, and the more detailed tables give separate figures for families consisting of 2 adults; 2 adults, 1 child; 2 adults, 2 children; and so on up to families with 2 adults, and 6 children.[1] (One would judge that families which included dependent adults were not brought into the investigation.)

In 1887 Ernst Engel proposed to elaborate a scale determining the "rank of consumers," based on the money value of the annual consumption of persons of different ages.[2] In 1891 when Wright, as United States Commissioner of Labor, was in charge of a study of the cost of living of workers in certain industries throughout the United States, a new method of measuring the demand for food was worked out. Under the heading "The consumption of food in normal families," the 1891 report [3] reads:

> It is evidently improper to compare family with family, for one family may consist of a husband and wife and one child, and another of a husband and wife and two, three, four or five children. Units of consumption have therefore been

[1] *Sixth Annual Report of the Bureau of Statistics of Labor*, Massachusetts (1875), p. 373.
[2] *Bulletin de l'Institut Internationale de Statistique*, Vol. II. p. 50.
[3] *Seventh Annual Report of the Commissioner of Labor* (United States), 1891, Vol. II, p. 859.

established simply for the purpose of comparison. Whether the units decided upon are perfectly accurate or not is not of so much importance as it is to secure a basis for comparison, but the units have been decided upon after mature consideration and consultation with various parties. . . .

Consuming Power Ascribed to Different Members of Families.

Members of Family	*Units of food consumption*
To husbands	100
To wives	90
To children 11 to 14 years, inclusive	90
To children 7 to 10 years, inclusive	75
To children 4 to 6 years, inclusive	40
To children 1 to 3 years, inclusive	15

The report does not say whether these units of comparison are based upon estimates of relative food costs, or relative energy requirements. The fact that the figures for children under seven years of age are so low is especially surprising if this scale is compared with that published by the bio-chemist, W. O. Atwater, in the *Seventeenth Annual Report of the Massachusetts Bureau of Statistics of Labor in 1886*.[1] Using the work of the German physiologist, Voit, and his associates, Atwater constructed a scale of the energy requirements of a laboring man at moderate work, a woman at "ordinary" work, children aged 6 to 15, children aged 2 to 6, and children one and a half years old and younger. He estimated the energy requirement of the youngest as 25 per cent of that of the adult male at moderate work and the group 2 to 6 years old as 50 per cent, as compared with the Federal Bureau of Labor figures of 15 per cent and 40 per cent for approximately the same age levels.

In 1895, Ernst Engel, Chief of the Royal Bureau of Statistics of Prussia, devised a scale by which the consumption of the individuals in different families might be rated.[2] He based his ratings primarily upon variations of individuals of different ages in weight per centimeter of height, making the child at birth 1, and the man of twenty-five 3.5.

Although Engel's scale is frequently quoted as something of a curiosity in the history of statistical measurements, it has not exercised as much influence in the United States as Wright's 1891 scale, which has been utilized in all the major investigations of cost of living in this country ever since it was first published. W. O. Atwater continued to refine his work on energy requirements, and published in 1901 a new scale which raised the relative requirement of the child under two years to thirty per cent of that of the man at moderately active muscular

[1] P. 239.

[2] *Bulletin de l'Institut Internationale de Statistique*, Vol. IX, Lebenkosten Belgischer Arbeiterfamilien.

work and established new categories, including the man at hard muscular work, with an energy requirement twenty per cent higher than that of the man at moderate work, but the United States Bureau of Labor used the 1891 scale unchanged in analyzing the cost of living data collected in its 1903 survey of cost of living.

In the succeeding years laboratory techniques for measuring basal metabolism were improved, and many controlled experiments were conducted for the purpose of measuring the metabolism of individuals under different circumstances. In 1917 the New York Association for Improving the Condition of the Poor published a survey by Lucy Gillett of the dietary investigations which had been carried on in the past twenty-five years, for the purpose of assisting persons who were actually engaged in measuring the food requirements of dependent families. In his introduction to the Gillett study H. C. Sherman says in part:

> In computing the food requirements of family groups, it has been customary to regard the man as the unit and assume that the food of each child may be represented by some appropriate fraction of the food of the father. This practice naturally arose from the fact that the food requirements of men had been longer studied and were better known than those of children, few investigations having been made upon children by accurate laboratory methods until within comparatively recent years. . . . It seems hardly logical to make this the basis for estimating the dietary needs of a family. Thus a carpenter may require 3,500 calories per day; a tailor, 2,500; a 14-year-old son of either of these men, 2,800 calories. With the carpenter as the unit, the boy's requirement would be represented by 0.8 of that of the father; but with the tailor as the unit, the allowance of 0.8 would obviously provide far too little food for the boy's needs. The Atwater dietary standards for children were stated as decimal fractions of the requirement of a man at moderately active muscular work, such as a carpenter requiring 3,400 or 3,500 calories.
>
> In practice, the dietitian who makes use of these decimal fractions in computing the food requirements of the family finds it difficult to avoid the tendency to reckon the child's food requirement according to that of the father, which works serious injustice to the child if the father have a low food requirement because of being engaged in an occupation which does not involve active muscular work.

Miss Gillett abandons the use of relative figures and gives the energy requirements of healthy children in calories, year by year, and separately for boys and girls.

Nevertheless, in 1919 when the United States Bureau of Labor Statistics analyzed the cost of living data collected in 1918–1919, the 1891 Wright scale was used without revision for measuring the demand for food among the 12,096 wage-earning families surveyed. An introductory statement reads as follows:

> Of course not all the members of a family consume the same quantity of food. Careful studies and comparisons of food consumption have lead to the assumption

that, as a rule, using the food consumed by an adult male (one 15 years of age or over) as a basis, or 1.00, the relative consumption of food is as follows:

and the scale published in 1891 is repeated![1]

If units for measuring the relative demand for food are to be improved, it would seem necessary that each investigator should state upon what basis his units are computed. To say that a scale represents relative "food consumption" is not to be specific. The food consumed by different individuals may be measured in terms of pounds, of calories, of protein, calcium, phosphorus or iron, or in terms of its money value. I am of the opinion that scales constructed on the basis of the money value of the food consumed by persons of different age, sex and occupation will be the most satisfactory for the purposes of students of family expenditures.

A scale based on relative money expenditures for food was constructed by Edgar Sydenstricker and W. I. King and published in this JOURNAL for September, 1921. In the course of a study of the expenditures of 1,500 South Carolina mill village families, figures on total earnings and total expenditures as well as on expenditures for food became available for each family. Each family was rated according to its energy requirements by means of the Atwater estimates of energy requirements published in 1902, and households were classified according to income groups at intervals of $4.00 monthly per Adult Male Unit. (The Adult Male Unit was defined as the need for heat and energy from food, to the extent of the average number of calories required to keep a normal young man who is engaged in moderate muscular work in a proper state of health and vigor. It would seem that it might be clearer to call this an "energy unit" rather than an Adult Male Unit, since the adult male has other characteristics besides a need for heat and energy, but Sydenstricker and King were simply following the Bureau of Labor Statistics nomenclature.) Each income class was dealt with separately and the food expense per week per "Adult Male Unit" was computed. It was found at first that the expense per unit showed a tendency to increase slightly as the percentage of females in the family increased. The tentative expenditure scale based on the Atwater energy scale was adjusted for this factor, and then for the age factor, and two scales (one for each sex) were finally arrived at which "purported to show the actual relative expenditures for food of persons of different sexes and various ages." The name chosen for the unit in which both scales are expressed is fammain—that is, food for adult male maintenance. "The fammain may be accurately defined for any given class of people *as a demand for food of a money value equal to that demanded by the aver-*

[1] U. S. Bureau of Labor Statistics, Bulletin No. 357, *Cost of Living in the United States*, p. 70.

age male in the given class at the age when his expense for food reaches a maximum." The authors of this study performed a valuable service in stressing the necessity for defining the basis upon which any scale for measuring the demand for food is constructed, and in pointing out the desirability of using a scale for each sex showing average relative total maintenance expenses for persons of different ages[1] in cost of living studies. An important part of such a scale must be a scale showing average relative expenses for food.

In discussing the fammain scale, the authors say that "it is evident that unless the food of females costs more per calorie than does that of males, or *vice versa*, or unless the cost of food per calorie varies with age, or *vice versa*, the calorie curve and the fammain curve must be identical." A study of the data from about twelve hundred families showed "no correlation whatever between either age or sex and the food cost per calorie. The conclusion must be that the fammain curve represents the demand for food equally well, whether it is expressed in calories or in money cost." [2]

This very correspondence between relative food costs and relative calorie consumption among the South Carolina families on whose expenditures the fammain scale is based detracts from its general usefulness.

Laboratory work on the nutritive requirements for animal growth begun about 1906 by Hopkins in Cambridge, England, and by Osborne and Mendel at New Haven, has since been carried on by so many workers and in so many places that there is now a large body of information on the relation of protein, minerals and vitamins, as well as calories, to human growth. The work on vitamins has not yet yielded any convenient methods for the accurate measurement of vitamin requirements. It is clear, however, that a very large proportion of foods rich in vitamins is necessary in the child's diet if healthy growth is to be obtained. In addition the fact that the calcium, phosphorus, and iron requirements of children are either absolutely or relatively higher than those of their elders, that their energy and iron requirements vary quite differently from their calcium and phosphorus requirements, and that protein requirements vary according to still another scale makes it impossible to construct any single index or even two indexes to measure food requirements satisfactorily, unless it be in terms of the money values of the different diets. A large proportion of the population now takes account of the "newer knowledge of nutrition," and the spread of baby clinics and of public health nursing has resulted in a marked

[1] U. S. Bureau of Labor Statistics, Bulletin No. 357, *Cost of Living in the United States*, p. 30.
[2] *Ibid.*, p. 36 ff.

change in child feeding throughout a large section of the country. For a great many American families it is no longer true that food costs per calorie do not vary with age, for the diet with a high vitamin and mineral content generally prescribed for younger children is more expensive per calorie than an adequate diet for the adult male at moderate work.

In a recent investigation of the food consumed by farm and village families in Central New York State,[1] an attempt was made to secure figures on the retail value of low-cost, but nutritionally adequate diets in accordance with the food habits of the people of that section of the country. Detailed records of food consumption over a period of four weeks were secured from 106 families. These records of food consumed were analyzed as to the adequacy of their protein, calcium, phosphorus, and iron content.[2] Adequacy of the calories consumed was judged according to the extent of the variation of the weight of the individuals in the family from the average for their age and height. An attempt was made to analyze the diets as to vitamin content, but it was abandoned because of the fact that there are many foods for which no rating of vitamin content has yet been made. This lack was in part remedied by the fact that the standards for mineral adequacy were very rigidly adhered to, and many of the foods rich in calcium, phosphorus or iron are sources of one or another of the vitamins. The families coöperating in the study were economically independent and in good health at the time of making the food consumption record. After the diet analyses were completed it was found that 18 (43 per cent) of the village families and 41 (64 per cent) of the farm families were adequately fed according to the nutritional standards adopted for the purpose of the study. In order to compare the food expenditures of the adequately fed families a scale was devised in terms of the money value of low-cost, adequate food-budgets for persons of different ages, sizes and occupations at prices prevailing in central New York villages.[3] This scale is based on the money value of an adequate diet for an adult male at moderately heavy muscular work requiring 3,400 calories a day. This value is taken as 1.00 and the values for other individuals are stated as percentages of this base. (See table at end of article.)

In preparing this scale, food budgets were compiled which would supply low-cost diets for persons with different food requirements,

[1] F. M. Williams and J. E. Lockwood, *An Economic Study of Food Consumed by Farm and Village Families in Central New York State*, Cornell University Experiment Station Bulletin.

[2] The writers had the assistance of Professor Mary F. Henry of the Department of Foods and Nutrition at Cornell University and Professor Henry F. Sherman at Columbia University in choosing the standards for nutritional adequacy.

[3] Janet Watson Bump was responsible for the calculation of these budgets.

thoroughly adequate according to the nutritional standards used in analyzing the food consumption records received, and in accordance with the food habits of adequately nourished families in that part of the country, as shown by the records. The money value of each food budget was figured by applying the average prices given in the food consumption records to the amounts of foods listed. Because of the wide variations among families in the amounts of food furnished by farm and garden, for the purpose of computing these relative money values, all the foods in each budget have been evaluated in terms of village retail prices.

The relatively high figures for young children in the Cornell adequate-food-cost scale are to be explained by the relatively high cost per 100 calories of young children's diets as compared with those of active adults. The annual retail value per 100 calories for the one-year-old child is $7.05, for the average adult male at moderate work requiring 3,400 calories $5.30, and for the average adult male at very heavy muscular work requiring 4,600 calories $4.89.

Whether the money relationships between these food budgets for persons of different ages and occupations would be appreciably different if they were evaluated at city instead of village prices has not yet been determined. It is interesting, however, to compare them with figures taken from the Chicago Standard budget quoted by Leila Houghteling in her study of Chicago wage-earners' families. Here the relative money value assigned to the children's diets is even higher. The food budgets on which the Chicago figures were computed are not available to the present writer. It may be that the high cost of "certified" milk in Chicago is responsible for the high figures given for children in the Chicago budgets.

In view of the wide divergencies in the scales shown in the appended table it seems important that some research on the measurement of the demand for food be a part of the next extensive cost of living investigation. Food costs for adults vary materially according to the energy required daily and, as far as I know we have had no classification of our population according to energy requirements. We do not know what proportion of adult males are engaged in "moderately heavy muscular work," in "heavy muscular work," and in "very heavy muscular work," as compared with those in sedentary occupations. There is need for more laboratory research into the nature of energy requirements of adults, particularly of women, in different occupations, but that is no reason for ignoring the laboratory work that has already been done.

When the statistical family used by the Bureau of Labor Statistics, husband, wife, boy of 12, girl of 6, and boy of 2, is rated according to the

Bureau of Labor scale, its demand for food is equivalent to that of 3.35 adult males. If the demand for food were to be computed according to the method in use at Cornell it would be necessary first to compute energy requirements, and therefore to know the weight of each member of the family and the occupation of the parents. If the father were a miner and weighed 165 pounds, his demand for food would be very different from what it would be if he weighed 150 pounds and were engaged in desk work. The addition of such details contributes greatly to accuracy and does not add materially to the labor involved in a cost of living survey. Supposing the father to be a man weighing about 165 pounds doing work equivalent to that of a carpenter, the woman to weigh about 135 pounds, doing all the housework for the family, and the children to be of average weight for their respective ages, according to the Cornell scale the family's food would cost 3.63 times that of the adult male at moderate work, almost 10 per cent higher than the Bureau's estimate. If the father were a miner (employed full time) the total demand for food of the family would be 16 per cent higher than the figure used by the Bureau which continues to assume "a certain equal consuming power for all husbands." [1]

[1] *Seventh Annual Report of the Commissioner of Labor*, Vol. II, p. 859.

MEASURES OF THE RELATIVE DEMAND FOR FOOD

	U. S. Bureau of Labor Statistics [1] (Adult Male Units)	Sydenstricker and King [2] (Fammains)	Relative energy requirements [3]	Chicago standard budget [6] (Relative Costs)	Cornell Adequate-food-cost units [7]
Adult male	100	1.000	....	....	....
Adult female	90	.858	....	....	....
Man at moderate work	...	....	1.00[4]	1.00	1.000[4]
Man at heavy work	...	....	1.12[4]	1.10	1.112[4]
Man at very heavy work	...	....	1.35[4]	....	1.249[4]
Woman at moderate work	...	....	.71[5]	.80	.775[5]
Woman at heavy work	...	....	.82[5]	.87	.848[5]
Boy 17 years	100	.935	1.00	1.12	1.027
Child 14 years	90	.769	.79	.95	.861
Child 11 years	90	.592	.71	.75	.783
Child 8 years	75	.506	.53	.65	.600
Child 5 years	40	.435	.44	.60	.520
Child 2 years	15	.348	.33	.60	.417
Child 1 year	15	.302	.28	.60	.372

[1] Food consumed by an adult male (one 15 years of age or over) = 100. *Cost of Living in the United States*, Bulletin 357, U. S. Bureau of Labor Statistics, p. 70.

[2] Demand for food *of a money value* equal to that demanded by the average male in a given class at the age when his expense for food reaches a maximum = 100. E. Sydenstricker, and W. I. King, "The Measurement of the Relative Economic Status of Families," this JOURNAL, September, 1921. Children's figures given above apply to boys.

[3] Energy requirement of the man at moderate work = 100. H. C. Sherman, *Chemistry of Nutrition*, p. 527; and M. S. Rose, *Laboratory Handbook of Dietetics*, p. 12.

[4] Weight taken as 165 lbs.

[5] Weight taken as 135 lbs.

[6] Cost of food of man at moderate work = 100. Florence Nesbitt, *The Chicago Standard Budget for Dependent Families*, Chicago Council of Social Agencies, Bulletin No. 5.

[7] Retail value of food budget for man at moderate work = 100. F. M. Williams and J. E. Lockwood, *An Economic Study of the Food Consumption of Farm and Village Families*, Cornell University Experiment Station Bulletin.

NOTES

THE GEOMETRIC MEAN OF PERCENTAGES

By Arthur F. Burns, *Rutgers University*

The geometric mean is deemed by statisticians an exceptionally "good" average to employ when percentages are being averaged. Academic sanction has led to an increasingly extensive utilization of the geometric mean for this purpose. It may be helpful, therefore, to point out several economies in the process of computing a geometric mean of percentages, and to discuss one serious limitation of the geometric mean in the determination of an average of percentages.

The arithmetic labor of establishing the geometric mean of a series of link ratios (or relatives) may be reduced considerably. The geometric mean in this case may be computed directly from the initial and terminal values of a given time series, for the geometric mean of a series of link ratios is equal to b in the exponential function $a_n = a_0 b^x$, where a_n and a_0 are the initial and terminal magnitudes, respectively, and x is the number of time units. This may be proved readily:

Let the successive time values be $a_0\ a_1, a_2, \ldots\ldots a_n$.

The link ratios will then be $\frac{a_1}{a_0}, \frac{a_2}{a_1}, \ldots\ldots\ldots \frac{a_n}{a_{n-1}}$.

If x is the number of link ratios, the log of the geometric average is

$$\frac{\log\left(\frac{a_1}{a_0}\right)+\log\left(\frac{a_2}{a_1}\right)+\ \ldots\ldots\ +\log\left(\frac{a_n}{a_{n-1}}\right)}{x}$$

$$=\frac{\log a_1-\log a_0+\log a_2-\log a_1+\ \ldots\ldots\ +\log a_n-\log a_{n-1}}{x}$$

$$=\frac{\log a_n-\log a_0}{x}.$$

But $\frac{\log a_n-\log a_0}{x}=\log b$ in the function $a_n=a_0b^x$.

This is self-evident if the latter expression is reduced to logarithmic form.

It is patent that a real time economy is achieved by employing the formula: log of geometric mean of link ratios $=\frac{\log a_n-\log a_0}{x}$. Looking up two logarithms and one antilog, and a single subtraction and division

are the arithmetic processes substituted for a long chain of computations—laborious even when mechanical aids are employed in the work of calculation. Needless to say, the longer the period of the series, the greater is the economy attained.

The geometric mean of a series of fixed base ratios (or relatives) of a given time series may be computed from the original values without first reducing the series to ratio (or relative) form. The formula to employ in this case is: log of geometric mean $= \frac{\Sigma(\log a) - x(\log a_0)}{x}$, where $\Sigma(\log a)$ refers to the sum of the logs of the original values, where a_0 is the base period (in this case, the initial) value, and x is the number of time units. The derivation of this formula is as follows:

Let the successive time values, as above, be $a_0, a_1, a_2, \ldots\ldots a_n$.

Then the fixed base ratios will be $\frac{a_0}{a_0}, \frac{a_1}{a_0}, \ldots\ldots \frac{a_n}{a_0}$.

If the number of time units is x, the log of the geometric mean is

$$\frac{\log\left(\frac{a_0}{a_0}\right)+\log\left(\frac{a_1}{a_0}\right)+\ldots\ldots+\log\left(\frac{a_n}{a_0}\right)}{x}$$

$$=\frac{\log a_0-\log a_0+\log a_1-\log a_0+\ldots\ldots+\log a_n-\log a_0}{x}$$

$$=\frac{\Sigma(\log a)-x(\log a_0)}{x} \text{ or}$$

$$\frac{(\log a_1+\log a_2+\ldots\ldots+\log a_n)-(x-1)\log a_0}{x}.$$

The essential economy that is attained by using this formula should be obvious: A single multiplication and subtraction are substituted for the series of divisions that the reduction of a given series to ratio (or relative) form necessitates. The formula given admits of general application. If, say, the tenth year of a series is the base year, the formula for that case becomes $\frac{\Sigma(\log a)-x(\log a_{10})}{x}$; and if the base is an average of a series of years, say, $\frac{a_6+a_7+a_8}{3}$, the formula is

$$\frac{\Sigma(\log a)-x\left[\log\left(\frac{a_6+a_7+a_8}{3}\right)\right]}{x}.$$

When the geometric mean is used, not in the case of a single series, as in the above two general instances, but for the purpose of combining into a single numeric value a group of disparate items, such as price or

production ratios (or relatives), a short-cut procedure is given by the formula: log of geometric mean $= \frac{\Sigma(\log a_1) - \Sigma(\log a_0)}{x}$, where $\Sigma(\log a_1)$ and $\Sigma(\log a_0)$ are the sums of the logs of the given and base year magnitudes, respectively, and x is the number of items. This expression may be derived by simplifying the formula customarily employed:

Let a'_1, a''_1, a'''_1, etc., be, say, the prices of a series of commodities in a given year; a'_0, a''_0, a'''_0, etc., the prices in the base year; and x the number of price items.

Then the price ratios will be $\frac{a'_1}{a'_0}$, $\frac{a''_1}{a''_0}$, $\frac{a'''_1}{a'''_0}$, etc.

The log of the geometric mean equals

$$\frac{\log\left(\frac{a'_1}{a'_0}\right)+\log\left(\frac{a''_1}{a''_0}\right)+\log\left(\frac{a'''_1}{a'''_0}\right)+ \ . \ . \ . \ . \ .}{x}$$

$$= \frac{\log a'_1 - \log a'_0 + \log a''_1 - \log a''_0 + \log a'''_1 - \log a'''_0 + \ . \ . \ .}{x}$$

$$= \frac{\Sigma(\log a_1) - \Sigma(\log a_0)}{x}.$$

If an index number for only a single year is computed, this method does not make for an appreciable reduction of effort, for although a series of divisions is eliminated, the number of logarithms that has to be looked up is twice as large, and another addition and subtraction are required. But, if index numbers are computed for a series of years, which is usually the case, the economy is substantial: The computation of the ratios (or relatives) is done away with completely, while the work of consulting log tables is practically the same (only one additional series being needed).

The above formula may be extended to include the weighted geometric mean. The derived formula is: $\frac{\Sigma(W \log a_1) - \Sigma(W \log a_0)}{\Sigma W}$, where the symbol W stands generically for the weights used. This formula is arrived at in the same manner as the formula for the simple geometric mean, and its derivation may, therefore, be omitted. The economies achieved in this case are substantially the same as those secured by using the short-cut method of computing the simple geometric mean. If a weighted geometric for only a single date is computed, a variant of the above formula is preferable, to wit:

$$\frac{\Sigma[W(\log a_1 - \log a_0)]}{\Sigma W}.$$

The latter is the only instance (and it is a special, not a general instance) of all the cases presented, where the margin of economy is rather narrow, if not fictitious, for though a group of divisions is eliminated, there is substituted for it a series of subtractions and the looking-up of a series of logarithms.

The primary purpose of this paper is to present a number of simplified methods for computing a geometric mean of percentages. Yet, it may be worth while to discuss briefly a point suggested by the fact that the geometric mean of a series of link ratios is the equivalent of b in the exponential function $a_n = a_0 b^x$. This point has an important bearing on the applicability of the geometric mean in the averaging of link ratios (or relatives).

It has been shown that the geometric mean of link ratios (or relatives) may be computed most readily by substituting for the initial and terminal magnitudes and for the period of the series, in the compound interest formula, and solving for the unknown, b. In using this method, the assumption is made implicitly that the initial and final magnitudes are "normal values"—*i. e.* that they lie approximately on the line of trend of the series. In the event that this assumption is invalid, interpolated values will not trace the "average" course of the original observations. The computed average percentage rate of change per given time unit may, then, contain an appreciable error, for it will reflect the "accidents" of the boundary dates rather than the "true" slope of the trend line. Even when the initial and terminal magnitudes *alone* are known, this method should not be used unless an analysis of the situation validates in some measure the assumption that the boundary values are "normal." Clearly, in the case of a continuous time series, all of the successive time values should be utilized in the determination of the average percentage rate of change. The geometric mean of successive percentage changes (of course, the computations would be carried out in terms of link ratios or relatives) seems to take the individual values into account. Yet, irrespective of what the internal contours of an historical series may be, provided the initial and final magnitudes are unchanged, the geometric mean of percentage changes will not vary at all. Very little significance, therefore, could be attached to a measure of this sort.

It appears that there is only one way of establishing for a given series an average percentage rate of change per unit of time that takes into account the internal structure of the historigram, and that is to fit, either by the method of least squares or by the use of Glover's "mean value table" or in some other way, an exponential function (logarithmic straight line) to the series. In that case, the antilog of b will be the de-

sired average of the link ratios, and the antilog of b minus unity the average percentage rate of change per given time unit.

It goes without saying that, if the initial and terminal magnitudes may be "justly" assumed to be "normal values," or if "satisfactory" visually chosen "normal values" may be substituted for the "irregular" boundary values, the solution for b in the compound interest formula will yield results tantamount to those obtained by fitting in a conventional way an exponential function to the observation points —and much more expeditiously. But in so doing, resort is necessarily had to an optical determination of "normal values." The element of arbitrariness so introduced could be ignored with profit if only a single computation were involved. As Dorothy C. Bacon has put it, "most statistical work of value depends on judgment to some extent." But in an investigation where numerous geometric averages of percentages may be calculated, the employment of a more "objective" technique— such as the fitting of exponential trends by the method of least squares —is to be preferred because it makes for greater comparability of the computed averages.

THE USE OF OVERLAPPING QUARTERLY INDEXES IN ERRATIC SERIES

By Donald S. Thompson and Robert W. Bachelor,
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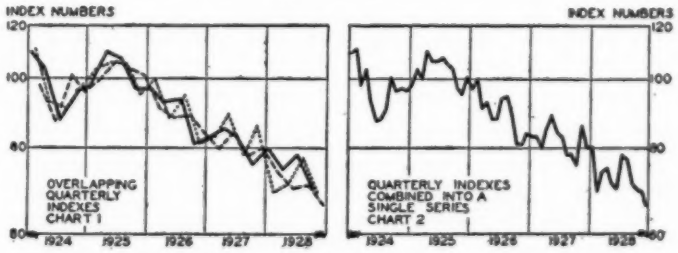
Series which have very erratic month-to-month fluctuations present a problem which is rather disturbing to the statistician.

This problem is generally sidestepped through smoothing the series after adjustment for seasonal variations by the use of a three-months' moving average, centered on the middle month, or more probably, in order to use the current figures in the smoothed index, upon the last month. Strict technical accuracy is properly sacrificed to expediency, the resulting error in most cases not exceeding the probable error of the original data.

In analyzing such irregular series relating to the Twelfth Federal Reserve District or series which have a seasonal which is apt to shift forward or backward one or more months, depending upon the weather or unknown factors of public taste or whim, the authors have found it desirable to make use of three overlapping quarterly indexes, so constructed that there is always an index ending on a particular calendar month. For example, an index is constructed for the quarterly total of November–December–January; another for the total of December–



January–February; a third for the January–February–March total; a fourth for February–March–April, etc.

The use of these indexes of quarterly totals secures a smoothed index, seasonally corrected, and free from certain objections to smoothing by means of a moving average. It permits the use of any standard method of measuring seasonal variations of a fixed or shifting nature. It is a much more accurate method of correcting an erratic series for seasonal fluctuations which are often difficult to determine by the usual procedures and the determination of which on a monthly basis is of doubtful validity. We also find that it represents a considerable saving of labor as compared with other methods.

The procedure briefly is as follows:

1. Construct three quarterly series for quarters ending: (a) March, June, September, December; (b) February, May, August, November; (c) January, April, July, October.

2. Adjust each quarterly series separately for seasonal variations. (Adjusted quarterly indexes of building permits, 1923–1925 average = 100, are shown in Chart I.)

3. Combine the resulting quarterly series into a single monthly series. (Chart II shows the three quarterly indexes plotted in Chart I combined into a single continuous monthly index series.)

We have applied this method to series such as building permits issued and tonnage of vessels built, which are noted for erratic and violent fluctuations of such a character that an index of monthly seasonality is of doubtful value. It has also been applied to quarterly totals of contracts awarded for engineering construction and to other series. These overlapping quarterly indexes have exhibited a very creditable behavior when compared with monthly indexes in spite of the difficulty of seasonal correction.

This method of smoothing erratic series facilitates forecasting the behavior of such series. Forecasts for the following year or longer may be made in terms of quarterly totals, one ending upon each calendar month. This effectually irons out erratic forward or backward fluctuations in seasonality and permits much greater accuracy in forecasting. It is particularly suited to forecasting monthly distribution of sales throughout the year, which distribution is likely to be disturbed by early or late seasons. In ordinary business forecasting the use of shifting quarters is sufficiently accurate to conform with usual accounting practices.

INTERLOCKING MEMBERSHIPS OF SOCIAL SCIENCE SOCIETIES

By Stuart A. Rice and Morris Green, *University of Pennsylvania*

In this paper we present tables showing the extent of overlapping among the membership lists of four social science societies. The figures provide indexes, first, of the degree to which the interests of each professional group are confined within its own organization; second, of the degree to which the interests of any two groups tend to coincide. Several specific questions which suggested the inquiry are of general interest, and at the same time illustrate, in the answers given, the utility which the data may have:

(1) The American Statistical Association at its annual meeting, December 28, 1928, amended its constitution to provide for the election of six vice-presidents, each representing "a distinct field of statistical interest." The obvious intent was to counteract noticeable tendencies for the Association to become too closely identified with a single field. How pronounced have these tendencies become? Reference to our data shows that among the statisticians, economists are more than six times as numerous as sociologists, and more than sixteen times as numerous as political scientists.

(2) The Social Statistics Committee of the American Statistical Association wishes to learn precisely the extent of common ground between statisticians and sociologists. We find that the number of persons who belong jointly to the American Statistical Association and the American Sociological Society is 74. Those having this combination of affiliations exclusively, however, number 33,[1] or 1.85 per cent of the membership of the former society and 2.43 per cent of the latter.

(3) One of the present writers is interested in the application of statistical methods to political science.[2] What basis for promoting the use of these methods is provided by the present membership of the American Political Science Association? It is found that 30 members of that Association, among 1,081, are likewise members of the American Statistical Association.

(4) The American Economic Association is generally regarded as the parent organization from which other social science societies have sprung. Does it have that priority among the secondary interests of members of the other societies that the historical relationship would suggest? The question is of importance in various ways, especially in

[1] Among the four societies here considered, 33 persons have affiliations with these two and no others.

[2] Stuart A. Rice, *Quantitative Methods in Politics*, A. A. Knopf and Company, 1928.

connection with the arrangements for the various annual meetings. The answer seems to be found in the fact that each of the other three organizations has its highest percentage of overlapping membership with the Economic group.

Membership lists for the four organizations as of the year 1928 were kindly provided by the secretaries.[1] Lists for other organizations were either unobtainable for 1928,[2] or were not regarded as of sufficient value to warrant the geometrically-increasing labor that each new list imposed, since each must be compared with all of the others. For the actual checking, the organizations were coded with numbers from 1 to 4, as follows:

American Sociological Society	1
American Statistical Association	2
American Economic Association	3
American Political Science Association	4

In the accompanying tables, these code numbers are replaced by abbreviations for the respective societies. Honorary members, foreigners in America and Americans abroad, non-member subscribers, and institutional members such as libraries, have in all cases been excluded from the computations. The distributions of members according to affiliations are given for each society in Tables I to IV.

TABLE I
OTHER AFFILIATIONS OF MEMBERS OF THE AMERICAN SOCIOLOGICAL SOCIETY[1]

	Number	Per cent
Affiliated with:		
No other affiliations	1,142	84.21
Statistics, exclusively	33	2.43
Economics, exclusively	98	7.23
Political Science, exclusively	23	1.70
Statistics and Economics	33	2.43
Statistics and Political Science	1	.07
Economics and Political Science	19	1.40
Statistics and Economics and Political Science	7	.52
Total membership	1,356	100.0
Summary of joint memberships, including duplications:		
Sociology and Statistics	74	5.46*
Sociology and Economics	157	11.58*
Sociology and Political Science	50	3.69*

[1] Among the four social science societies here considered, only. We have no data concerning affiliations with other societies not included in the study.
* Based upon 1356, the total membership.

[1] That for the American Political Science Association is mimeographed as of February, 1929. That for the American Sociological Society is printed in the Society's *Publications*, Vol. XXII, 1928, and includes all names up to date of publication in June, 1928. That for the American Economic Association is contained in the latter's *Handbook*, published as a Supplement to *The American Economic Review*, Vol. XVIII, No. 2, June, 1928. That for the American Statistical Association is contained in its *Handbook*, published as a Supplement to this JOURNAL, Vol. XXIII, New Series 164, December, 1928. The dates of the four lists seem sufficiently close to give comparability.

[2] Thus, no list for the American Historical Association, which we wished to include, was procurable since the supplement to the *American Historical Review*, Vol. XXX, No. 3, April, 1925.

TABLE II

OTHER AFFILIATIONS OF MEMBERS OF THE AMERICAN STATISTICAL ASSOCIATION [1]

	Number	Per cent
Affiliated with:		
No other affiliations	1,249	69.89
Sociology, exclusively	33	1.85
Economics, exclusively	442	24.73
Political Science, exclusively	5	.28
Sociology and Economics	33	1.85
Sociology and Political Science	1	.06
Economics and Political Science	17	.95
Sociology, Economics and Political Science	7	.39
Total membership	1,787	100.0
Summary of joint memberships, including duplications:		
Statistics and Sociology	74	4.14*
Statistics and Economics	499	27.92*
Statistics and Political Science	30	1.68*

* Based upon 1787, the total membership.

TABLE III

OTHER AFFILIATIONS OF MEMBERS OF THE AMERICAN ECONOMIC ASSOCIATION [1]

	Number	Per cent
Affiliated with:		
No other affiliations	1,851	72.82
Sociology, exclusively	98	3.86
Statistics, exclusively	442	17.39
Political Science, exclusively	75	2.95
Sociology and Statistics	33	1.30
Sociology and Political Science	19	.75
Statistics and Political Science	17	.67
Sociology, Statistics and Political Science	7	.28
Total membership	2,542	100.0
Summary of joint memberships, including duplications:		
Economics and Sociology	157	6.18*
Economics and Statistics	499	19.63*
Economics and Political Science	118	4.64*

* Based upon 2542, the total membership.

[1] Among the four social science societies here considered, only. We have no data concerning affiliations with other societies not included in the study.

In Table V, "Exclusiveness" is used to designate the extent to which the membership of each society is confined to that organization alone, among the four here considered. This characteristic is presented in the order in which columns have been arranged from left to right, and is indicated by the percentages in the first row.

Several of our conclusions have been presented at the outset. We might remark, in addition, upon the evidence presented by the tables of the degree to which economists have been interesting themselves in statistical problems, as compared with sociologists and political scientists. Statistics may fairly be regarded as a science having to do with *tools* rather than with particular types of social relationships, as in the

TABLE IV

OTHER AFFILIATIONS OF MEMBERS OF THE AMERICAN POLITICAL SCIENCE ASSOCIATION[1]

	Number	Per cent
Affiliated with:		
No other affiliation	934	86.40
Sociology, exclusively	23	2.13
Statistics, exclusively	5	.46
Economics, exclusively	75	6.94
Sociology and Statistics	1	.09
Sociology and Economics	19	1.76
Statistics and Economics	17	1.57
Sociology, Statistics and Economics	7	.65
Total membership	1,081	100.0
Summary of joint memberships, including duplications:		
Political Science and Sociology	50	4.63*
Political Science and Statistics	30	2.78*
Political Science and Economics	118	10.92*

* Based upon 1081, the total membership.

[1] Among the four social science societies here considered, only. We have no data concerning affiliations with other societies not included in the study.

TABLE V

MUTUAL "EXCLUSIVENESS" OF FOUR SOCIAL SCIENCE SOCIETIES

	Political Science	Sociology	Economics	Statistics
Percentage of given society belonging to no other	86.40	84.21	72.82	69.89
Percentage belonging to one other society	9.53	11.36	24.20	26.86
Percentage belonging to two other societies	3.42	3.90	2.72	2.86
Percentage belonging to three other societies	.65	.52	.28	.39
Total	100.0	100.0	100.0	100.0

case of the other three "disciplines." Nearly 70 per cent of the statisticians are without other affiliations among the other three societies. Biologists, psychologists and educational research specialists may account for a portion of this number, but we suspect that the larger number are primarily interested in business enterprises, and hence to be classed rather with the economists than with the other professional groups. If so, this would still further widen the comparison just made.

Another point which might be made, is that while the sociologists and political scientists are about equal, proportionately, in the extent to which they share memberships with the economists, those who extend this interest to statistics are proportionately more numerous among the sociologists. However, this is probably carrying inference farther than the data warrant, and we prefer to have other conclusions drawn by the reader as he sees best.

FINANCING THE NATION'S TRADE AND INDUSTRY

A dinner meeting of the American Statistical Association was held at the Aldine Club, 200 Fifth Avenue, New York City, on Thursday evening, May 9, 1929. Two hundred and seventy-nine persons were present. Dr. Edmund E. Day of the Rockefeller Foundation presided. The general topic for discussion was, "Financing the Nation's Trade and Industry."

The first speaker of the evening was Carl Snyder, Chief Statistician of the Federal Reserve Bank of New York. The subject of his address was, "Long Time Trend of Industry and Credit Requirements." The speaker began by pointing out the general impressions that have prevailed at the time of the great business depressions of the past. He showed that, when a depression is on, residents of financial centers usually feel that business throughout the nation has almost ceased to function. Such was the view in 1873 and again in 1893. Mr. Snyder, however, presented a series of charts showing that most of the leading indicators of business activity, when the annual data are plotted on ratio charts, follow approximately straight lines, and show an average increase at about the rate of 4 per cent per annum, and even the most severe business depressions, when viewed in retrospect, are found to represent only slight deviations from the trend. It appears, then, that industry is not nearly so unstable as most students of the subject formerly believed. The reason an erroneous view in this regard is so widely held, is that many students of the business cycle have given too much attention to such indicators as pig-iron production, which are far from being representative of the course of business activity as a whole.

The persistence of a constant trend is in some cases remarkable. For example, one would never believe, without examining the facts, that the production of gas would have climbed upward along approximately a straight line, despite the advent of electricity. The movement of volume of postal receipts, likewise, has varied but slightly from a straight line, even though rates have been changed frequently and business has fluctuated.

There has been a marked tendency during the whole of the last half century for production to increase much faster than the rate of growth of population. The idea that a marked expansion in per capita output is characteristic only of the period since 1921, is, then, distinctly erroneous.

Mr. Snyder emphasized the fact that, while production has increased rather steadily at approximately the rate of 4 per cent per annum, bank credit has not expanded regularly at this rate, but, at times, has grown much more rapidly than production. Whenever this has occurred, the price level has risen. There is very close correspondence between the deviations of the volume of bank credit from a straight line trend and the deviations of the *general* price level from its trend. This correspondence has been overlooked in the past, largely because those making comparisons have commonly used indexes representing the prices of commodities at wholesale. Such indexes are far from typifying the general price level and, hence, it is not surprising that one finds little evidence of correspondence between their movements and the movements of the curve representing the volume of credit.

The second speaker of the evening was Professor Oliver M. W. Sprague of Harvard University. It was his view that, although it may be granted that an increase of 4 per cent per annum in bank credit is the normal requirement and although this normal requirement may serve, in a broad way, to tell something about the extent to which credit should be advanced, it is not in itself an adequate guide for the regulation of the volume of bank credit. In such regulation, it is necessary to take account of more specific factors. For example, in recent months, stock prices have moved sharply upward while the general price level has remained on a fairly stationary level. Under these circumstances, it may well be that an attempt should be made to control security prices even though the course of other prices were as an incident to be unfavorably affected. It may be necessary also to take specific action to prevent or alleviate industrial depressions, and such specific action may not be the same as that which would be indicated by a regular 4 per cent increase in bank credit.

It is well to remember that the immediate destination of bank credit is not necessarily the same as its ultimate goal. For example, a large proportion of the funds entering the stock market eventually arrives in general business, although the action of the stock market doubtless modifies the flow to some extent.

The movements of prices and gold supply are not always readily responsive to changes in discount rates. India, for example, absorbs each year a considerable proportion of the world's gold output and she does this without much regard to the status of the price level or bank credit. The demands of stock speculators for credit at times seem to be impervious to any ordinary movements in the discount rate. One cannot, therefore, expect to control stock speculation by adjusting the Federal Reserve Bank re-discount rate by a fraction of a per cent.

If the Reserve Banks are to exercise control over speculation, it is, therefore, necessary for them to take action of a definite and conspicuous character, calculated to influence the speculative temper of the community and so the demand for credit. They must, however, be careful not to go too far. It would, for example, be out of the question to insist that all banks must immediately eliminate loans on stock exchange collateral. Direct action, aimed at persistent borrowers at Reserve Banks, is an appropriate policy, but to cope with a general credit problem such as that presented by widespread speculation, main reliance must be placed upon the discount note.

Many writers have contended that, if the Federal Reserve discount rates were advanced sharply, business would be penalized. As a matter of fact, there is little evidence that advances in interest rates alone are often responsible for business declines. It is true that it is not unusual for a collapse of business to accompany a rise of interest rates, but this is far from proving that it is caused by the rise of interest rates. The rise in interest rates frequently indicates exhaustion of the credit supply, and this exhaustion of the credit supply is likely to depress business.

If the Federal Reserve Banks, by energetic use of the discount rate, were to curtail sharply the demand for credit used by the stock market, business, in the long run, would gain more by the elimination of this disturbing factor than it would lose by any temporary increase in the re-discount rate.

The third speaker of the evening was Professor B. Haggott Beckhart of Columbia University. He treated of fluctuations in bank credit through 1927–1929 relating these to the open-market operations of the Federal Reserve Banks. The significance of these operations consists in their effect on the reserve balances of member banks upon which the superstructure of bank credit is based. In the second half of 1927 the Reserve Banks increased their holdings of United States securities by 225 million dollars with a resultant increase in member-bank reserve balances and an increase in the total loans and investments of the reporting member banks of 1,306 million dollars. In the first half of 1928, the sales of securities by the Reserve Banks checked the expansion in credit, which increased again rapidly in the second half when the Reserve Banks reversed their open-market policy and increased rapidly their holdings of bankers' acceptances. Since the first of 1929, holdings of bankers' acceptances have been reduced sharply, with a parallel decline in member-bank reserve balances and in bank credit as measured by the loans and investments of reporting member banks. The funds released through open-market operations in this period must be considered in conjunction with other factors that influence the member-bank reserve position, such as gold movements, earmarkings, currency fluctuations, and changes in member-bank borrowings, in the Federal Reserve "float," and in the amount of government deposits at the Reserve Banks, etc.

Taking into account these qualifying factors, there is no question but that fluctuations in the open-market holdings of bills and securities by the Reserve Banks, have induced parallel fluctuations in member-bank reserve balances and in the total volume of bank credit.

The discussion of the program was begun by Professor Wesley C. Mitchell of Columbia University. He called attention to the fact that the three speakers of the evening had treated the general topic of discussion from radically different points of view. He referred to Mr. Snyder as a cosmological statistician—that is, one interested in the broad sweep of events. Professor Sprague, on the other hand, represented the type of man interested primarily in what ought to be done. Professor Beckhart characterized the scientist with a microscope examining with great care the details of activity in a very limited period.

The discussion was continued by Colonel Malcolm C. Rorty of the International Telephone and Telegraph Company. In referring to the fact noted by Mr. Snyder that the volume of bank credit tends to parallel the course of the general price level, Colonel Rorty stated that this was almost necessarily true because of the fact that a rise in the price level compels loans and discounts to increase in proportion. The fact should, however, be noted that a decline in the price level does not necessarily pull bank credit down as rapidly as an increase in the price level forces it up. The speaker also felt that it was almost certain that any volume of bank credit which happens to be available will eventually be used up by business.

In commenting on Professor Sprague's desire for credit control, Colonel Rorty expressed the view that a suspension of the usury laws of New York State would make it much easier to curtail bank credit than it is under existing legal regula-

tions. It is by no means universally possible for a bank to know what is done with the money which it loans.

An interesting recent development is the tendency of the Federal Reserve Banks to lose control of the credit market. This situation has come about because of the fact that non-banking corporations are now furnishing a large part of the credit used in the stock market as call loans. Frequently, the money secured by the sale of stocks is immediately loaned again in the market. This tends to increase the price of stocks and the proceeds from sales of stocks at these higher figures may in turn be loaned on call. Theoretically, and to a considerable extent practically, this cycle may continue indefinitely.

One need have but little fear that the stock market situation will not remedy itself. As soon as people conclude that stocks are not going higher, the demand for excessive stock market credit will rapidly disappear. Recently, the general public has overestimated the value of common shares, just as it previously undervalued them.

Mr. James F. Hughes of Otis and Company, who closed the discussion, believed that Professor Sprague was right in holding that the adequacy of the supply of credit is more important than the interest rate. However, the same truth holds as regards the stock market. It is not at all probable that if the Federal Reserve Banks should increase their re-discount rate to 6 per cent, the effect upon stock prices would be at all marked.

Mr. Hughes remarked on the peculiar fact that, during the last eight years, despite the tremendous expansion in the volume of loans on securities, there has been practically no growth in the volume of commercial credit issued by banks. A study by Mr. Blakemore, also of Otis and Company, shows what became of 929 million dollars of stock subscribed to by stockholders of 72 companies in 1928. Subscribers to this stock were all stockholders of companies whose stock was listed on the New York Stock Exchange. Of the total sum, 746 millions were not used for refunding operations. Of this latter quantity, however, only 132 millions went into actual construction work of the companies securing the money, 128 millions was invested in the securities of subsidiary companies. Some of this may have gone into construction. The outstanding fact is that 423 millions, or 57 per cent of the amount not used for refunding operations, went into improvement of working capital positions. Of this amount, 23 millions was used to reduce bank loans and 9 millions went to increase the amount of cash on hand. The destination of the remaining amount could not be definitely ascertained. There is, however, little doubt but that a considerable portion of this remainder was utilized as brokers' loans.

One should not be too ready in condemning corporations for entering the loaning field. They have been educated to this by the banking fraternity. Investment bankers have found that securities can be floated with profit to themselves and, hence, they have encouraged the issuance of new securities. Commercial bankers were instrumental in educating corporations in the method of using surplus funds in the call market.

It has been said that the Federal Reserve System is one part of the Government not subject to checks and balances. It appears, however, that, at present, loans

by private corporations are serving to limit materially the power of the Federal Reserve Banks.

The meeting adjourned.

WILLFORD I. KING

PROGRESS OF WORK IN THE CENSUS BUREAU

THE CENSUS ACT

The Act providing for the Fifteenth and subsequent decennial censuses was signed by President Hoover on the 18th of June. Therefore the Bureau of the Census, if it may be personified, now knows where it stands and what it has to do. It has to take a census of population, agriculture, irrigation, drainage, distribution, unemployment, and mines in the year 1930 and tabulate and publish the results by January 1, 1933. It may be noted that the subject of manufactures does not appear in this list, but it is provided for by the biennial census of manufactures which will be taken in regular course in 1930 covering the year 1929.

The census of population and of agriculture is to be taken as of the first day of April. The provision for taking the census in November—which was in the bill at one time—was rejected partly because many Congressmen with rural constituencies contended that the rural population would not be adequately represented if the census were taken in the fall of the year.

APPORTIONMENT OF REPRESENTATIVES

The Census Act contains a section relating to the apportionment of Representatives. It provides that after each census the President shall transmit to Congress a statement showing the population of the several states as ascertained by the census and also the number of Representatives to which each state will be entitled under an apportionment of the then existing number of Representatives (at present 435) by each of the following methods:

(1) By the method used in the last preceding apportionment; (2) by the method known as the method of major fractions, which happens to be the same as (1); and (3) by the method known as the method of equal proportions. Then if the Congress to which this statement is submitted fails to pass a law apportioning Representatives, the apportionment by method (1) as announced by the President goes into effect. In short, this section of the law provides for what may be called an automatic apportionment in case Congress fails to act. If, however, the statement required from the President is not submitted within one week after Congress meets in regular session, these provisions of the law have no effect. So unless the Bureau of the Census is to bear the onus of nullifying the law, it must have its count of population completed and apportionment tables ready by the time Congress meets in December following the date of the census.

These provisions as to the apportionment of Representatives encountered vigorous opposition in Congress, which was the main cause of the delay in the enactment of the census law. This is the first time since the passage of the

act providing for the Census of 1850 that a census act has contained a section regulating the apportionment of Representatives.

THE CENSUS OF DISTRIBUTION

It may be noted that two new subjects have been added to the decennial census by the Act of June 18, namely, distribution and unemployment. Careful consideration is being given to the organization of these inquiries; and conferences are being held and advisory committees organized with the view to obtaining the best judgment and advice that the Bureau can secure.

The test census of distribution taken in eleven cities in 1927 developed important facts in regard to the sales of commodities by retailers, wholesalers, brokers, and similar agencies. No effort was made at that census, however, to include data relating to the sale of commodities by one manufacturer to another for use in assembling or for the purpose of fabricating other products. To determine the practicability of including industrial distribution in the national census, a canvass was made in Cleveland, Ohio, in coöperation with the Domestic Commerce Division of the Bureau of Foreign and Domestic Commerce. The questionnaire used called for the kind and cost of raw materials, semi-finished products and completely manufactured parts, mill supplies and fuel, purchased and used by manufacturing establishments in Cleveland in fabricating products for sale, as well as the kind and cost of materials, equipment and supplies purchased for use in plant maintenance, extensions to plant and buildings, and plant operation. The questionnaire also included an inquiry showing the kind and volume of products sold by class of buyers, as follows: (1) Sold to wholesalers, jobbers, etc.; (2) sold to chain stores; (3) sold to retailers; (4) sold direct to consumers; and (5) sold to miscellaneous purchasers.

In coöperation with the Domestic Commerce Division of the Bureau of Foreign and Domestic Commerce and the Louisville Grocery Survey, a census of food outlets was made in Louisville, Kentucky.

The results of these censuses and the experience gained in making the canvasses will be instructive and helpful in formulating plans for the national census.

In cities the distribution census will have no connection with the census of population, but will be taken under a separate organization by special agents employed for that purpose; and the work will probably be started about the 1st of February. It is probable, however, that the data regarding the country store will be collected by the regular census enumerator at the same time that he canvasses the rural districts for the population and agricultural census.

THE CENSUS OF UNEMPLOYMENT

It is generally realized that it is not going to be an altogether easy matter to get a reliable census of the unemployed. The data must be obtained in connection with or as a part of the population census. The first question is how shall we define unemployment; and having agreed upon our definition, how can we insure a complete and accurate return of "unemployment" as thus defined. It has been suggested that an unemployed person may be defined as one who is (a)

usually employed for wages or a salary, (b) out of a job, (c) able to work, and (d) looking for a job.

It is proposed to obtain this information by means of a supplementary schedule on which the enumerator will make an entry for every gainful worker who for any reason is not working at the time of the census. The questions carried on this schedule will develop the reason for his not being at work. If he is out of a job he will be asked how long that has been the case. If he has a job and is nevertheless not at work, he will be asked whether or not he is drawing pay—as he might be doing, for example, if he were on vacation or on sick leave. It is hoped that the tabulation of the cases where a man has a job, but is not drawing pay, will afford some index of the extent of part-time unemployment.

Once the unemployed have been identified, all of the data on the general population schedule will be available for correlation, so that they may readily be classified by color, sex, age, marital condition, occupation, etc., so far as worth while.

PREPARATIONS FOR THE GENERAL CENSUS

In the meantime the Bureau is going forward with its preparations for the general census. The country has been divided into approximately 575 supervisors' districts; and applications for the position of supervisor are being received daily. Under the law the supervisor is appointed without civil service examination by the Director of the Census. Undoubtedly many persons make application for this position without much idea as to what the duties of the office are and perhaps without much disposition to learn. The Bureau, however, is taking precautions to guard against the appointment of incompetent and uninformed supervisors. On receiving an application the applicant is sent a copy of the general instructions to supervisors, a pamphlet of about 50 pages. He is requested to read this and then to notify the Bureau whether he still wishes to be considered an applicant for the position. If he does he is sent an application blank, together with a test schedule which he is required to fill out and return with his application. This is the same kind of test that is given to enumerators. The facts that are to be entered on the schedule are stated in the form of a descriptive narrative and it is safe to say that no one could fill out the schedule correctly without having studied the instructions. So far as practicable the Director, or some other representative of the Bureau, will visit the supervisors in advance of the census and instruct them orally as to their duties.

The preparatory work incident to the establishment of enumeration districts is approaching completion, over 80,000 such districts having been established out of a total of approximately 110,000. For the cities of 100,000 population this work necessitated the numbering and describing of over 320,000 city blocks, a job of considerable detail, but of great importance in the administration of census-taking in those places where the population is especially congested and it is difficult to make a complete enumeration.

While the law provides that the enumerators, like the supervisors, shall be appointed by the Director of the Census, it provides also that this authority may be delegated to the supervisors; and in practice that is what will be done in most

cases. All applications for appointment as enumerator, however, must be filed in Washington, and the test schedules as filled out by the applicants will also be sent to Washington where they will be marked and graded. The Bureau will then send to each supervisor a list of those applicants within his district who have successfully passed the test, from which list he must make his selections. The Bureau without any advertising has already received about 50,000 applications for appointment.

J. A. H.

UNEMPLOYMENT INQUIRY IN THE FIFTEENTH CENSUS

Now that legislative provision has been made for an inquiry into unemployment in the forthcoming census of population, the Committee on Governmental Labor Statistics has been negotiating further with the census authorities in the matter.

On May 28, Mr. Edward E. Hunt of the Department of Commerce and Mr. Leon E. Truesdell of the Bureau of the Census met with representatives of the Committee and others invited to confer with them on the subject. The conference was held in the offices of the Social Science Research Council in New York. Those present were Mr. Edward E. Hunt, Mr. Leon E. Truesdell, Professor Wesley C. Mitchell, Dr. Leo Wolman of the National Bureau of Economic Research, Mr. Meredith Givens of the Social Science Research Council, Professor Robert E. Chaddock and Dr. Willford I. King, statistician members of the Advisory Committee of the Census, Miss Mary van Kleeck and Messrs. William A. Berridge, Ralph G. Hurlin and Bryce M. Stewart from the Committee on Governmental Labor Statistics. After a lengthy discussion of a number of proposed questions on unemployment, Mr. Truesdell asked that the Committee on Governmental Labor Statistics draw up its final recommendations and submit them to the Director of the Census. In view of this request, a subcommittee consisting of Miss van Kleeck, chairman of the Committee, and Messrs. William A. Berridge, Ralph G. Hurlin, Eugene B. Patton and Bryce M. Stewart drafted a number of questions and forwarded them to the Director of the Census.

It will be noted that the questions submitted by the Committee apply to the week before that in which the census will begin, rather than to May 1, 1930, the beginning date, or any other date, and also that the Committee seeks to arrive at the degree of unemployment by inquiring what proportion of a full week's pay was earned. The Committee fears that if the questions refer to unemployment on a particular date the answers in many instances will not indicate the individual's actual employment status. For example, many railway employees, having their rest day on the census date, would answer that they were not working or that they were unemployed on that date and would be counted as unemployed. On the other hand, such persons would indicate that they earned a full week's pay if the question were asked, and would be properly counted as employed.

The Committee also considers it necessary to segregate persons whose jobs are continuously on a part-time basis, as for example certain waiters and salespeople who are employed only during rush hours. The Committee feels that it is desir-

able that the information should be related to occupations. Some of those enumerated will be attached to or separated from jobs in their usual occupations. Others will be attached to or separated from jobs not in their usual occupations. In the Committee's view an effort should be made not only to determine the state of employment and unemployment as related to the occupational conditions of the week of the census but also to determine the nature of shift in occupation, if at all possible. This is an especially difficult problem and the questions on this point are submitted only as a basis for discussion. The Committee assumed that the questions will be asked of all those in the class of gainful workers, with no limitations as to sex or age, and that elsewhere in the enumeration distinction will be made between the self-employed and those working for others. The questions submitted are as follows:

Usual occupation

1. Trade, profession or particular kind of work, as *spinner, salesman, riveter, etc.*
2. Industry or business, as *cotton mill, dry goods store, ship yard, etc.*

Present, or last occupation if unattached to a job

3. Trade, profession or particular kind of work, as *spinner, salesman, riveter, etc.*
4. Industry or business, as *cotton mill, dry goods store, ship yard, etc.*

Employment and unemployment of gainful workers

5. Did person hold a job of any kind during the week ended Saturday, April 26, 1930?
6. If job was held, was it regularly on part-time basis?
7. Did person earn a full week's pay, part of a week's pay, or no pay during that week?
8. If person held no job or if he received part of a week's pay or no pay on job during that week, was the cause
   (a) Illness or accident?
   (b) Vacations, holidays or other personal reasons?
   (c) Strike or lockout?
   (d) Lack of work due to weather?
   (e) Slack work?

INSTRUCTIONS TO ENUMERATORS

Question 5

Whether person held job. Write "Yes, usual," "Yes, makeshift," or "No." "Yes, usual," if person holds job at his ordinary occupation, "Yes, makeshift," if person holds job at some occupation not his usual occupation and plans some day to return to his usual occupation. Write "Yes, usual," or "Yes, makeshift," if person is working for self, if person is attached to job but on temporary layoff, if person is attached to job but absent on account of illness, vacation, etc., if person did not work or worked only part of the week because of existing strike or lockout.

Question 6

This question is designed to cover part-time waitresses, part-time office cleaners, part-time saleswomen, etc., whose positions never permit a full week's work. Persons such as certain railway employees who have rest periods between shifts of work are, of course, not to be considered part-time workers. *Do not ask further questions of part-time workers.*

Question 7

Pay. Write "F" for full week's pay, "P" for part of a week's pay, and "N" for no pay. If person earned overtime pay that week, assume he earned a full week's pay.

Question 8

Cause of no pay or part of a week's pay. Write

"I" for Illness or accident.

"V" for Vacations, holidays, or other personal reasons.

"D" for Strike or lockout (dispute).

"W" for Lack of work due to weather.

"Sl" for Slack work.

If there were other causes operating, do not check.

If more than one of the causes listed were operative, indicate each operating cause.

(a) covers illness or accident of self and others. (e) covers those working only part of a week because of slack work on job and those who were entirely separated from their jobs because of slack work.

MISCELLANEOUS NOTES

The San Francisco Chapter.—The San Francisco Chapter's third meeting of this year was held on the evening of May 10. The general topic of the evening, "Statistics and the Stock Market," assisted by the continued increase in membership and strength of the organization drew a record attendance of more than 50 members and guests.

The first speaker, Professor W. L. Crum, discussed the subject "A Statistical Description of the Present Speculative Situation," dealing for the most part with the chief characteristics of pertinent statistical data at present available and discussing them from the point of view of statistical theory and economic considerations. His remarks may be summarized under five headings as follows:

1. Prices. Although the present price level of stocks appears to be high, available composite representations afford uncertain bases of comparison of the present with the past because of variations in the rates of change of prices of different stocks, dissimilar group changes and other causes. It is possible, perhaps, to speak of the value of individual stocks with greater assurance. Even here, however, it is difficult to obtain entirely satisfactory data on the book value or equity of corporation shares, and available data on a corporation's earning power are necessarily based on the past, whereas the proximately present and future earning power should have an equal, if not greater influence on prices of its stock. Furthermore, large individual business organizations and different lines of industry are not so specialized today as formerly. Industry is becoming more and more a hodge podge with a resulting increase in the difficulties of accurately ascertaining trends of earning power and equity values.

2. Activity of Trading. From the information available it is not easy to secure a satisfactory indication of this important factor in the present speculative situation. Total volume figures are to be had, and they indicate very great activity. But, in themselves, these figures supply us with no knowledge of the preponderance or lack of preponderance of specific individual issues or groups of issues involved in the trading. The identity of traders usually has an important bearing on the activity of trading and the speculative situation. The only generally available information on this subject at present comes in the form of rumors.

3. Securities Issues and Credit. Securities markets are affected by the issuance of new securities. New issues of both bonds and stocks have been in large volume during recent years. The issuance and sale of new stocks have distributed proprietary interests at the prevailing high, or at least advancing, prices and have been highly speculative. The proceeds of bond sales in the past generally have been sunk in capital investment. In a sense such use of funds is speculative, for returns on the investment at its present capitalized value are dependent to a large extent on the maintenance of present profits margins. Furthermore, approximately one-fifth of recent new issues represent the paper of investment trusts. These stocks are not a direct proprietary interest in productive enterprise, but are merely the paper representatives of still other paper. They are not, therefore, productive in the economic sense. So far as they serve to raise the stated total of new securities issued they give a false sense of the amount of new capital made available to industry. In addition to these factors, there is a large, varying, and probably indeterminable amount of credit tied up in so-called undigested issues of securities, a fact which further complicates the use of credit data in any statistical analysis of the speculative situation.

4. Credit. An abundance of data concerning the credit situation has been made available during recent years. So far as it concerns bank loans it is good. But securities markets are influenced also by the volume of credit and funds involved in so-called bootleg loans and in advances by brokers on margin accounts. Accurate information on these two phases of the credit situation is not available and it probably is well-nigh impossible of development.

5. Conclusions. On the whole, the picture presented by a statistical description of the present securities markets indicates highly active speculation and the presence of considerable inflation. This is true despite the imperfections and inadequacies of the data we now have. It is noticeable that speculation is not apparent in the grain or other commodity markets, in real estate or in foreign exchange. The speculative appetite of the American public apparently is finding its satisfaction almost entirely in the stock market.

The inadequacies of the material available for a statistical description of the present speculative situation have been stressed. Constantly changing conditions which alter the meaning of data with the passing of time and cause imperfect time comparisons affect the entire range of available information. Considerable lack of detail concerning the present situation of both individual concerns and lines of industry prevails, although in many instances information is available fairly soon after the close of the period to which it refers. Many pertinent aspects of the speculative situation, such as accurate knowledge of the amount of credit and funds tied up in undigested securities, are not covered by available data. It would be extremely difficult, if not impossible, to obtain some of these data because of the physical obstacles alone which would be involved. For example, it would seem impossible to devise a system of accounting by means of which we could accurately designate the amount of credit involved in carrying undigested securities and in advances on margin accounts. These

imperfections and inadequacies would seem definitely to forestall the construction or compilation of reliable statistical summariz :tions of the speculative situation.

Professor N. J. Silberling discussed "Trends and Cycles in Stock Prices." Three aspects of the characteristic behavior of common stock prices were discussed, the long-time changes, the major cyclical movements, and the minor or intermediate movements.

It is very difficult to measure accurately the gradual appreciation in stock values which can be imagined to be justified by the growth in corporate equities. We can estimate trends by statistical methods when dealing with the volume of production in established lines of industry; but in dealing with complex variables like the prices of common stocks, subject to very irregular cyclical movements, the last phase being still uncompleted, we cannot by these simpler methods determine a "normal base line" from which to measure deviations. Reference was made to the factors of a political nature which appear to have restrained speculative enthusiasm in the decade from 1908 to 1918, roughly. To this fact the more than ordinary increase in values since the War may be at least partially attributed. The divergent results given by available index numbers of stock prices as to the degree of change in the level of the leading issues on the New York market during the last ten years is striking.

The next topic concerned the nature of the pattern of the major cyclical movements, which was illustrated by a chart of monthly average prices, based on six market-wise stocks, from 1917 to April, 1929. Since 1924 there has been no close relationship between the cyclical movements of the market and either interest rates or indexes of business activity and industrial production. These relationships are again complex in their nature and require special interpretations and statistical methods to bring them out. For instance, there is a consistent relation between fundamental credit conditions and the major movements in prices if we convert statistics showing the available supply of credit into a cumulative form. The law of the major cycles seems to be that so long as credit is available at rates within reason, the public will bid up stocks, and so long as credit supply is below its normal level there will be a continuous downward tendency in the market. The fluctuations in credit are much less closely related to stock prices than to the position of the credit supply with reference to its long-term normal which may be taken to indicate a dividing line between plentiful and depleted credit supplies and between abundant corporate balances with moderate commercial borrowing strain and the opposite situation of depleted balances and heavy reliance on the banks for short-term assistance.

With regard to the relation of the major movements to internal factors in the operations and financial results of the several corporations, and the general ups and downs of business conditions, it has been found that the stocks which regularly move in closest unison with the market as a whole are those of companies whose business operations move closest to the general business cycle, even though the pattern of these cycles is not in recent years similar to the average major tendency of the stock market. Again, there are special groups of stocks which move through major cycles having a distinctive pattern which is related to the fluctuations in the prices of certain commodities, such as sugar, oil, leather, or copper. In these cases, however, the special group patterns are always composites, in which the movements of the market as a whole play an important part.

These considerations suggest the powerful effect of the major movements on the more market-wise individual issues. Individual stocks respond, of course, to developments (or prospective developments) in the earnings, dividends, and accumulated surpluses of corporations, but frequently these are offset and neutralized by the force

of the general price tendency. The market is a great fabric in which the threads are mutually reënforced and respond to the influences affecting the entire fabric.

In discussing, finally, the shorter minor movements in the market, it was pointed out that the market does not move in a perfectly continuous manner through the major swings, but accomplishes its objective by a halting progress up or down, with many interruptions and small reversals of direction. These movements, which are of great interest from both a statistical and a practical point of view (because of the large possibilities of profit in market operations which they provide) resemble the major movements in the fact that they do not correlate consistently with any single economic or financial series. Interest rates are usually supposed to be an important factor in generating these minor fluctuations, but close study of this relationship, with the use of deviations of time money rates from a moving base, reveals no consistent correlation between either the actual or the cumulated money rates and the prices.

In fact the minor movements represent mainly the play of public enthusiasm and the operations of pool interests. Both of these factors are reflected in the volume of trading, but the fact that the amount of the trading assumes different meanings at various stages of each minor cycle makes it difficult to develop any simple statistical series for correlation purposes based on the volume of stock sales. The most perfect results in the interpretation and forecasting of the short-term movements of this type are based on close analysis of the price momentum and step by step scrutiny of the trading with an understanding of the probable meaning of each change in the volume of sales, rather than by rigid correlations for regular lead and lag.

Meetings of the Chicago Group.—The Chicago Group has had three meetings since those described in the last issue. On April 20 Mr. Carl Snyder of the Federal Reserve Bank of New York, and former President of the American Statistical Association, addressed the Group on the subject "Business Stability and Business Cycles." Mr. Snyder discussed long-time economic trends and the relative insignificance of the business cycle in comparison with those fundamental trend movements. Attendance was more than 100, the best in the history of the chapter or of its precedessor, The Society of Business Analysis.

The meeting of April 30, under the chairmanship of Mr. Franklyn Hobbs, Director of Research, Central Trust Company, was addressed by Mr. Glenn Griswold, Editor of the *Chicago Journal of Commerce*. Mr. Griswold presented his views regarding methods whereby statistics and statisticians can be of more service to modern business, regarding the possibilities of coördinating the many sources of business statistics, and regarding other problems, the solution of which will benefit the business statistician.

Mr. S. W. Russell, of Swift and Company was the chairman of the meeting held on May 21. The speakers were Mr. H. M. Fleming of Eastman, Dillon and Company, subject "Applications of Statistics in Forecasting Stock Prices"; Mr. R. B. King of the Illinois Bell Telephone Company, subject "Special Statistical Methods Used in Forecasting Telephone Calls"; and Mr. H. G. Pett of the Federal Reserve Bank of Chicago, subject "Scope and Future Development of Statistics in the Federal Reserve Banks."

At this meeting the following were elected as officers for the year 1929–1930: President, G. L. Seaton, Commercial Engineer, Illinois Bell Telephone Company; Vice-President, H. G. Pett, Manager, Department of Research and Statistics, Federal Reserve Bank; Secretary-Treasurer, R. B. King, Statistician, Illinois Bell Telephone

Company; Directors of the Chapter in addition to the above officers, Franklyn Hobbs, Director of Research, Central Trust Company; Roscoe Smith, Thomson and McKinnon; H. E. Adams, Statistician, Kiwanis International.

There are now about 160 names on the mailing list, of whom about half paid dues to the local chapter during the last year. The attendance at meetings averaged about 60 during the year.

The Cleveland Chapter.—The fifth annual meeting of the Cleveland members of the American Statistical Association and the second annual meeting of the formal Cleveland Chapter was held at the Allerton House on Monday evening, April 29. There were 90 members and guests present at dinner. Colonel Ayres was the presiding officer.

In the business meeting preceding the program the following officers were elected: President, Mr. H. B. Flinkers, of the Federal Reserve Bank of Cleveland; and Vice-President, Mr. John W. Love, of the *Cleveland Plain Dealer*. For the dual office of Chapter and District Secretary the Chapter nominated Mr. D. C. Elliott, of the Midland Bank.

There were five papers presented at this meeting: "Adventures in Educational Research by Statistical Methods" by Mr. W. L. Connor, of the Cleveland Board of Education; "A Weekly Index of Industrial Activity" by Miss Lydia H. Barton, of the Vaness Company; "A Geographical Difference in the Seasonal Incidence of Typhoid Fever" by Dr. G. E. Harmon, of Western Reserve Medical School; "Statistics for the Man on the Job as Well as for the Executive" by Mr. Paul E. Harris, of the Ohio Bell Telephone Company; and "Security Prices and Federal Reserve Policy," by Colonel Leonard P. Ayres, of the Cleveland Trust Company.

At the close of the first year of its formal organization the Cleveland Chapter has a membership of 43, 26 active and 17 associate members.

The Twin City Chapter.—The Twin City Chapter of the American Statistical Association is still in process of organization. Letters and questionnaires designed to discover the wishes of the members of the Association in the Twin City area with reference to a local organization have been prepared and are going forward this summer. The first meeting for organization purposes will probably not be held until September or October. The group promises to be small but active, and to embrace several fields of statistical interest.

Activities of the Los Angeles Chapter.—The Los Angeles Chapter held a dinner meeting at the University Club on May 22, at which Dr. Keith Powelson of the Claremont Colleges presented a paper on "Profits of National Banks." Sixteen members and guests were present.

A dinner meeting was also held on June 28 at the Beverly Hills Club to welcome new members that have been taken into the Chapter and to promote friendship between the entire group. Swimming, volley ball, and cards were the program for the evening.

New members recently elected to the Los Angeles Chapter are Dr. Horace Secrist, who is on leave from the Northwestern University for two years' work with the Economics Commission of Claremont Colleges, Dr. Keith Powelson of Claremont Colleges, Mr. Glenn Wingard of the Research Department of Security-First National Trust and Savings Bank, Mr. H. A. Perryman of the Los Angeles Railway, and Mr. Robert S. Hopkins of Dowell and Hull.

Activities of the Austin Chapter.—The Austin Chapter held its last regular meeting of the academic year on May 15 with 27 members and guests present. Dr. E. L. Dodd, Professor of Actuarial Mathematics, the University of Texas, and President of the local Chapter, spoke on "How Great Must a Difference in Sizes or Measurements Be to Indicate Essential Differences in Individuals or Populations Measured?" Officers for the local Chapter were all reëlected as follows: President, Dr. Edward L. Dodd, The University of Texas; Vice-President, Mr. H. H. Schutz, Statistician for Texas, The United States Department of Agriculture; Secretary, Mr. C. D. Simmons, The University of Texas.

Meeting of the Pittsburgh Group.—On Friday, May 17, William F. Ogburn, of the University of Chicago, addressed a joint dinner meeting of the Pittsburgh group of the American Statistical Association and the Social Science Seminar of the University of Pittsburgh. This meeting was held at the Faculty Club on the University of Pittsburgh campus. Professor Ogburn discussed "The Statistics of American Marriage and the Family." About 30 people attended the meeting and participated in the discussion following Professor Ogburn's talk.

United States Bureau of Labor Statistics.—The study of the use and hazards of radium and radioactive substances in industrial processes was completed during the past quarter and the results published in the *Monthly Labor Review* for June. Also, during the quarter, a compilation of money wages in foreign countries was made by the Bureau of Labor Statistics, and at the request of Senator Shortridge was published as Senate Document No. 9, 71st Congress, 1st session. A reprint of the material in this document will be issued as a regular Bureau bulletin.

The monthly volume of employment surveys of the Bureau continued to be increased rapidly by the addition of new firms and also by the addition of new industries. The latest industry to be added is that of canning and preserving.

Beginning with July the Bureau took over from the Metropolitan Life Insurance Company the monthly compilation of labor turnover statistics. The work will be carried on in much the same manner as formerly, but with the hope of gradually extending its scope so that ultimately turnover figures can be published by industries and smaller geographical units.

Other recent activities of the Bureau have been concerned chiefly with wages and hours surveys. The annual survey of union wage rates is nearing completion, and wage studies of the following industries are in press or under way: Lumber, boots and shoes, men's clothing, hosiery and underwear, automobiles, coal, iron and steel, oil, and foundries and machine shops.

Coöperative Undertakings of the Federal Bureau of Agricultural Economics.—The Bureau of Agricultural Economics and the Department of Marketing (School of Accounts and Finances), New York University, are coöperating in a study of the price of oranges. The object of the study is to determine and measure the several factors which influence the establishment of price levels for oranges.

Another coöperative project, undertaken by the Bureau and the Connecticut Agricultural College, is a study of tobacco prices.

Activities of the Dominion Bureau of Statistics.—During the summer vacation, Professor Maxwell, of King's College, N. S., Professor L. W. Moffit, of Wesley College, Winnipeg, Man., and Mr. James Culliton, of McGill University, Montreal, P. Q.,

are working in the Dominion Bureau of Statistics, Ottawa, on special statistical problems, including the classification of occupations and the "origins and destination" problem in trade statistics. Professor W. B. Hurd of Brandon College is also working in the Bureau on a series of demographic maps for the use of the Canadian Pioneer Problems Committee and for incorporation in an atlas of agriculture and population to be issued subsequently.

Elections to the Social Science Research Council.—The Social Science Research Council has elected the following members under the recently created classification of Members-at-Large: Professor Henry M. Bates, Dean of the Law School of the University of Michigan; Dr. Adolf Meyer, Professor of Psychiatry and Director, Henry Phipps Psychiatric Clinic, Johns Hopkins University; Dr. C. E. A. Winslow, Graduate School Chairman, Professor of Public Health, Yale University; and Professor Robert S. Woodworth, Department of Psychology, Columbia University.

Grants and Fellowships Offered by the American Council of Learned Societies.—The American Council of Learned Societies announces that in March, 1930, a limited number of grants in aid of research, available at once, and of post doctoral fellowships, available from July, 1930, will be offered in the following fields:

Philosophy
Philology (all branches)
Linguistics
Literature
Art
Archaeology
Musicology
Oriental Studies (Sinology, Indology, Semitics, Assyriology, Egyptology)
History and auxiliary sciences (except those branches that are essentially economic, social, and political history), *e. g.*, history of religion, thought, science, learning and the fine arts, palaeography, diplomatics, chronology, etc.

The grants in aid of research are of two categories:

(A) *Small grants.* These grants are normally of the maximum of $300, but they may be increased to $500 when, in the judgment of the Committee on Aid to Research, such increase is justified by unusual expenses, as of foreign travel. The small grants are available to scholars of all ages (except for assistance in fulfilling the requirements for any academic degree) who are citizens of the United States or permanently employed or domiciled therein, and who are engaged in specific projects of research for which aid is actually needed and not obtainable from any other source.

(B) *Larger grants.* These range in amount from $750 to $2,000, and are reserved for mature scholars of demonstrated ability who are engaged in important enterprises of research to which they are able to devote at least six months without interruption. The same conditions as to citizenship, residence, or employment obtain as for the small grants. The object of the larger grants is to assist in the advancement of knowledge through aiding individual undertakings of fundamental importance.

The fellowships are of the post doctoral type already offered by the Social Science Research Council and the National Research Council. Their object is to assist the better training of research and teaching personnel, and they are limited to scholars, citizens of the United States, who have received the Ph.D. or have acquired its equivalent, and who are still in the "training period." The amounts of the fellowships will be determined by individual needs.

Applications for both grants and fellowships must be made by January 15, 1930, on special forms that will be furnished after October 1, 1929, by the Permanent Secretary of the American Council of Learned Societies, 907 Fifteenth Street, N. W., Washington, D. C., to whom preliminary inquiries may be addressed.

At the same time, attention is called to the grants-in-aid and fellowships offered by the Social Science Research Council, New York Central Building, 230 Park Avenue, New York City, in economics, statistics, political science, sociology, anthropology, psychology, social, economic and political history and related subjects, such as law and human geography.

Committee on Recent Economic Changes.—An outstanding achievement in statistical endeavor is the nation-wide inquiry carried on under the direction of the National Bureau of Economic Research which was the basis of the report of the Committee on Recent Economic Changes of the President's Unemployment Conference which was made public on May 15.

The research activities of the National Bureau in behalf of the Committee began in January, 1928, and lasted over a year, the Committee having completed its examination of the data in February of this year. Over a hundred educators, business executives, farm and labor leaders, economists, engineers and other scientific authorities, either individually or in connection with universities, colleges, various private and governmental fact-finding and other agencies, collaborated with the experts of the National Bureau in gathering data for the Committee.

Dr. Edwin F. Gay, of Harvard University, and Dr. Wesley C. Mitchell, of Columbia University, as directors of the research staff of the National Bureau, were in charge of the investigation which covered a great variety of subjects pertaining to economic conditions in the United States from 1922 to 1929.

The report of the Committee and the findings of the National Bureau have been published by the McGraw-Hill Book Company under the title *Recent Economic Changes*.

The Committee on Recent Economic Changes is made up of President Hoover, Chairman, Walter F. Brown, Renick W. Dunlap, William Green, Julius Klein, John S. Lawrence, George McFadden, Max Mason, Adolph C. Miller, Lewis E. Pierson, John J. Raskob, Arch W. Shaw, Louis J. Taber, Daniel Willard, Clarence M. Woolley, Owen D. Young, and Edward Eyre Hunt, Secretary.

Pennsylvania's Industrial Safety Campaign.—The Pennsylvania Department of Labor and Industry is promoting a state-wide industrial safety campaign for the year 1929. The campaign has developed into the greatest organized effort for industrial safety in the history of the Department. Some extraordinary features of the campaign as outlined by Harry D. Immel, Director of the Bureau of Inspection of the Pennsylvania Department of Labor and Industry, who is directing the campaign, are the following:

1. The signing of individual safety pledge cards by more than 2,500,000 workers.
2. The signing of more than 5,000 group pledge cards by plant executives showing plants pledged 100 per cent to safety.
3. The distribution of 250,000 lapel pins to workers bearing the official insignia of the safety campaign.
4. The furnishing of a series of safety posters to industrial concerns.
5. The furnishing of report forms to plants for the maintenance of a daily accident record and for the report of individual accidents by foremen to the plant manager or safety supervisor.

6. The establishment or reëstablishment of safety organizations in more than 1,000 plants.
7. The provision of a speakers' bureau service for safety meetings and radio broadcasting.
8. The provision of seven 200-feet trailer films for exhibition in moving picture houses throughout the State.

Visits of inspectors to industrial establishments are based on the accident experience of individual establishments as compiled by the Bureau of Statistics. Each month the supervising inspector of each of the nine inspection districts is furnished with a statistical report of accidents for each plant in his respective district which reported one or more accidents to the Bureau of Workmen's Compensation during the month preceding. The immediate concentration of inspection activities upon plants having bad accident experiences is thus made possible.

While a decrease in accidents has not resulted at the end of six months' intensive and unparalleled safety effort, it is reasonably certain that the 8.6 per cent increase in accidents for the first six months in 1929 over the corresponding period in 1928 is largely the result of increased exposure to accidents during the latter period. The more general reporting of accidents since the beginning of the campaign also is a factor in the accident increase. One undisputed achievement of the campaign thus far is that industrial workers in Pennsylvania have been educated in safety to a greater degree than ever before, and that the average industrial worker to a large extent has become safety-minded.

The Harris Memorial Foundation Institute on Population and Migration.—The Norman Wait Harris Memorial Foundation held its Sixth Institute at the University of Chicago, from June 17 to June 27. The meetings were devoted to the discussion of problems of population and migration. The Institute was attended by more than fifty investigators in this field of national and international reputation and furnished an opportunity for authoritative presentations of the present status of population research and for discussion of controversial questions. There were eight lectures open without charge to the general public, delivered by the following speakers:

Corrado Gini, Professor of Economics at the University of Rome, Italy; Director of the Central Statistical Bureau, Rome; author, Editor-in-Chief of *Metron;* and member of the Fascisti Grand Council.

Shiroshi Nasu, Professor of Rural Economics, Imperial University of Tokyo, Komaba, Tokyo, Japan; writer on Japanese food and population problems.

Robert R. Kuczynski, Professor of Economics, Berlin Handelshochschule; formerly Director of the Municipal Statistical Office, Berlin-Schoenberg; Council Member, Institute of Economics, Washington, D. C.

Oliver E. Baker, Economic Geographer, Division of Land Economics, Bureau of Agricultural Economics, U. S. Department of Agriculture; authority on climate and geography in relation to agriculture.

The Institute plans to publish these lectures in book form as it has done in the past.

During the Institute, Round Tables consisting of a limited number of invited guests with a special interest in the subject were organized to discuss certain phases of the general problem. Among those from out of the city who attended were Professor Raymond Pearl, Professor Charles B. Davenport, Professor Robert E. Chaddock, Professor Warren S. Thompson, Professor Ellsworth Huntington, Dr. Isaiah Bowman, Professor Edward A. Ross, Mrs. Margaret Sanger, Professor Walter Willcox and Professor Alonzo E. Taylor.

The Philadelphia Census Committee.—The Philadelphia Census Committee was organized for the purpose of securing more complete social and economic statistics for the city of Philadelphia. It is composed of representatives from corporations, city departments, and social agencies with city-wide interests. One of the first tasks of the Committee will be to secure census data for areas of smaller size than those now used for the publications of the Bureau of the Census. It is believed that basic population data of this type will greatly facilitate the research activities of the Department of Sociology at the University of Pennsylvania. Mr. A. B. Bishop, of the Bell Telephone Company, is chairman of the Committee, and Mr. W. Wallace Weaver, of the Department of Sociology at the University of Pennsylvania, is secretary.

The Brookings Institution.—The Brookings Institution has announced the appointment of the following fellows for the coming academic year: James A. Barnes, on leave of absence from the University of South Dakota; Gilbert L. Belsley, of the United States Bureau of Efficiency; William M. Blaisdell, of the faculty of Swarthmore; Chung-Fu Chang, of the Graduate School of Johns Hopkins; Lewis Corey, of New York City; Lora L. Deere, of Radcliffe College; Winifred Frost, of the faculty of Sweet Briar College; Erich Kraemer, of the University of Hamburg; Joseph S. Lawrence, of the faculty of Princeton; Yau Sing Leong, of the Columbia Graduate School; Fausto R. Pitigliani, from the University of Rome; Nathan Reich, of the Columbia Graduate School; Huntley Sinclair, of the faculty of Washington University; Herman Stoker, of the Graduate School of Cornell; Carl B. Swisher, reappointment; Kenneth O. Warner, of the faculty of the University of Washington; Virgil Willitt, of the faculty of Ohio State University.

The trustees of the Institution have appointed Edwin G. Nourse to the position of Director of the Institute of Economics, to succeed Harold G. Moulton. During the past academic year this position has been filled by Mr. Moulton as well as that of President of the Brookings Institution. Mr. Arthur C. Millspaugh, formerly Administrator General of the Finances of Persia, and Financial Advisor-General Receiver of Haiti, has accepted appointment with the Institute for Government Research of the Institution.

A Publication of the Carnegie Institution.—There has recently been published by the Carnegie Institution of Washington a volume on *Race Crossing in Jamaica*, the product of the Institution's Department of Genetics. This includes some 359 tables that are, for the most part, summaries of statistics concerning physical, physiological and mental traits measured by Dr. Morris Steggerda during twelve months at Jamaica, B. W. I. There were measured about 105 blacks, 165 browns and 100 whites, all largely of the agricultural class, males and females in nearly equal proportions. Results of over 50 physical measurements and indices derived from them are tabulated and distributions given in graphs. Over 1400 school children, ages 7 to 15, were measured. A few families were studied genealogically and anthropometrically. Among the large tables are Nos. 134–140, giving individual finger patterns for 351 persons; Tables 151 and 152, giving palmar configurations of 186 persons; Table 196, findings on basal metabolism of 50 blacks and browns. The color groups differ in mental traits just as they do in physical traits. The text of the volume was written chiefly by C. B. Davenport.

PERSONAL NOTES

Julius Vargha

The following letter has been received from the President of the Hungarian Statistical Society.

"BUDAPEST, May 27, 1929

"The Hungarian Statistical Society expresses deep sorrow that its popular and esteemed honorary member, Dr. Julius Vargha, late Secretary of State for Commerce, and Vice-President of the Hungarian Academy of Sciences, has died, at the age of 76. Julius Vargha was one of the leaders of Hungarian scientific and literary life; he spent more than four decades in the official statistical service. His work was always characterized by great knowledge, dauntless zeal and ardent patriotism. In the years 1892–1901 as Vice Director, and in 1901–1914 as Director of the Royal Hungarian Central Bureau of Statistics he played an important rôle in international statistical life, and was honoured by election to membership in the Royal Statistical Society, in 1904.

"In the name of the Hungarian Statistical Society.

"(Signed) Dr. GUSTAVE THIRRING,
President"

Dr. Vargha was also an honorary member of many years' standing of the American Statistical Association. His death is a serious loss to the statistical world.

Mr. Edgar Sydenstricker, Director of Research of the Milbank Memorial Fund, has returned from Geneva where he has been serving for several weeks as an advisor to the International Health Organization of the League of Nations on its statistical and epidemiological activities. In this capacity, also, he attended the meetings of the Commission of Expert Statisticians of the League in Berlin on April 3–6, and of the Mixed Commission of the League and of the International Institute of Statistics at Paris on April 9–12.

Mr. Sydenstricker has been appointed lecturer in Social Research (Public Health) in the Department of Social Sciences at Columbia University. He will conduct a seminar on methods of social research for graduate students during the session of 1929–1930. Two fellowships in social research have been provided by the Milbank Memorial Fund for the coming session at Columbia University.

William Leonard Crum, Professor of Statistics, Graduate School of Business, Stanford University, has been elected Professor of Economics in Harvard College, to serve from September 1, 1929.

Dr. Harold Hotelling of Stanford University is spending six months in England, working in mathematical statistics and mathematical economics. At present he is at the Rothamsted Experimental Station at Harpenden. He expects to return to California in January.

Dr. Meredith B. Givens has been made a member of the staff of the Social Science Research Council, giving special attention to the field of labor and industry. Recently with the U. S. Bureau of Labor Statistics he was engaged in a study of productivity changes in the iron and steel industry, and he has been associated with several studies under the supervision of the National Bureau of Economic Research.

Morris A. Copeland, who has been with the Division of Research and Statistics of the Federal Reserve Board, is returning to Cornell in September as Professor of Economics. Mr. George W. Blattner, who until recently was manager of the service department of the Bank of Commerce in New York City, and Mr. Frank A. Garfield, who is on leave of absence from the University of North Carolina, have become members of the Division of Research and Statistics. Other recent additions are Mr. Corliss E. Armstrong and Miss Margaret Ellis.

James W. Nixon has recently been appointed Chief of the Statistical Section of the International Labour Office of the League of Nations.

Dr. J. F. Ebersole of the Section of Financial and Economic Research of the Treasury Department, Washington, D. C., is going to Harvard University at the beginning of the fall term.

Mr. H. G. Pett, for several years a member of the Division of Research and Statistics of the Federal Reserve Bank of Chicago, was made Manager of the Division, effective July 1, 1929.

Mr. Benjamin P. Chambers became a member of the statistical organization of Selected Industries, Inc., an investment trust, on July 15.

MEMBERS ADDED SINCE JUNE, 1929

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REVIEWS

Die Saisonschwankungen als Problem der Konjunkturforschung, by Dr. Otto Donner. Viertel jahrshefte zur Konjunkturforschung, Sonderheft 6. Berlin: Reimar Hobbing. 1928. 66 pp.

Dr. Donner's treatise on the problem of measuring and eliminating seasonal factors in economic statistics is undoubtedly one of the most thorough discussions of the subject which has yet appeared. The author is of the staff of the Institut für Konjunkturforschung in Berlin. He is interested primarily in making a critical survey of the various methods of seasonal manipulation and passes in orderly and admirably presented review practically every method which the literature of the subject embodied at the time of his writing. The discussion is valuable as a comprehensive reference manual on seasonal methods for the experienced statistical worker, but is not suited to the purposes of the beginner.

After pointing out briefly the various kinds of circumstances which occasion seasonality in economic processes, the author discusses in a general way the properties of the several possible types of average which are applicable to a seasonal measure. The persistence of a certain amount of irregularity and of extreme values in almost every series from which seasonals are to be derived throws doubts upon arithmetic means. Modal values are usually restricted by the shortness of most series. The use of the median by Persons and others is conditionally commended, with the added observation that better results are obtainable by a modified median, *i. e.* an arithmetic average of the cluster of points about the median. The possibility of arithmetic or geometric averaging of all but the most extreme items in the arrays is also pointed out. There is an interesting treatment of the technique of testing the representativeness of seasonal indices by means of the dispersion about the central values obtained and the use of the quartile deviation.

A detailed examination is then conducted into the various methods, which are first broadly classified into those resulting in *fixed* seasonal norms and those resulting in *variable* seasonals.

Among the former methods that of "monthly means" is first treated and the generally accepted criticism is passed upon it of being deficient because of complications introduced by trend and cyclical influence upon the actual data. While the trend influence can be independently dealt with, the other factors remain to introduce errors.

The so-called Hall-Falkner method, using deviations from a trend rather than actual items, is shown to have shortcomings in requiring a very long period for satisfactory results.

The method of deviations from a moving average through the original items, which has been employed by the Federal Reserve Board, Macaulay, and Bowley, among others, is commended as a reasonably reliable one.

Persons' link-relative method is given somewhat more detailed attention, with special emphasis upon the possible errors arising from the "correction" of the chained medians for supposed trend influence. Donner shows the possibility of considerable discrepancies arising at the December-January point in series having practically no long-time trend, but where care has not been exercised in selecting perfectly typical mean values. As between the arithmetic and geometric corrections he favors the arithmetic. The link-relative and deviations-from-moving-average methods are ranked together as the most satisfactory methods of attaining fixed indices.

To obtain variable seasonal indices, allowing for progressive shift of seasonality in the data, there are again several possible methods. The method of Snow, applying trends to the actuals of Januaries, Februaries, etc., is not commended because of the danger of erratic items unduly influencing these trends. Gressens' device meets the same criticism. The methods of Crum and Flinn, applying either trends fitted by mathematical equations or smoothing lines based on moving medians (extrapolated either way) are considered, with a preference for the latter as being less likely to be affected by irregularities distorting the month-trends. It would seem that the author exaggerates this danger and overlooks the advantage of a straight line to the month-trends as better representing the true character of a gradual and progressive shifting of seasonal incidence. King's suggestion to sketch in the cyclical pattern by eye and work with the resulting deviations in deriving a measure of shift in seasonality is dismissed as being too much at the mercy of personal equation. In concluding this portion of his paper the author gives a numerical illustration of the great discrepancy between the results obtained by the fixed and the variable methods respectively on the same series. It would have been helpful to include a somewhat more detailed study of the difficulties which arise in relatively short series from the *assumption* of shifting seasonality where in fact there is merely the play of cyclical and erratic factors. The assumption of shifting can easily lead to manipulations of the material which in extreme cases have the effect of cutting out much of the cycle influence.

The reviewer hopes that Dr. Donner will apply his patient and acute analysis to some problems of seasonal measurement which were not compassed in this study. Among these are the problem of dealing with seasonality in compound series, built up of several variables each of which may contain seasonal influence of greater or less complexity, such for example as series of total employment in factories; the problem, again, of dealing with the highly "irregular" seasonal influence which emerges in processes connected with agriculture or shipping. Both of these problems still present considerable material for examination, testing, and comparison by the economic statistician. And incidentally the hope can be entertained that we may have at no distant date systematic compilations by the authoritative research bureaus of the seasonal norms which apply to the leading lines of economic activity in various countries.

NORMAN J. SILBERLING

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The Balance of Births and Deaths, Volume I, Western and Northern Europe, by Robert R. Kuczynski. (Brookings Institution Investigations in Population Statistics.) New York: The Macmillan Company. 1928. xi, 140 pp.

The birth-rate per head and the death-rate per head, taken by themselves, give an inadequate indication of the extent to which fertility and mortality are balanced in a given population, because both birth-rate and death-rate are markedly affected by the prevailing age distribution. The author, therefore, computes the ratio of the total births in two successive generations (which he terms the "net reproduction rate"), as determined by the age schedule of maternity frequency of the females and their life table. The process is carried out numerically for a number of European countries, with the following results (estimated for 1926, on the basis of latest available computations): Finland, 1.09; Denmark, 1.10; Sweden, 0.95; France, 0.94; Germany, 0.89; England and Wales, 0.88; all Western and Northern Europe, 0.93. That is to say, for this section of Europe, as a whole, each generation of births, at the present rates of female fertility and mortality, is by 7 per cent smaller than the preceding generation.

These are some of the outstanding results of the very interesting study made by Kuczynski. The obvious conclusion, at first sight, seems to be that the population of Europe is headed for a decline in numbers, despite the present excess of birth-rate over death-rate, an excess not due to inherent fertility, but to a peculiarity of the prevailing age distribution. The argument is identical with that of Dublin and Lotka's study, "The True Rate of Natural Increase,"[1] which is mentioned by Kuczynski in two incidental footnotes, in Chapters II and III without, however, giving the reader any indication that this prior study fully brings out very prominently the four principles set forth in italics as the very backbone of Kuczynski's Chapter IV, namely (1) *In the present population . . . the proportion of women in childbearing ages is particularly large and old persons particularly small.* (2) *The decisive factor in shaping the present population . . . was the trend of births.* (3) *The age composition . . . tends to lower the number of deaths.* (4) *The age composition tends to swell the number of births.* No reference whatever is made, throughout this chapter, to prior publication of these four principles, and the reader not acquainted with the literature will certainly be left with the impression that these principles were originated by the author. As they are fundamental to the whole thesis, the omission seems regrettable.

The following cautions must be observed in construing Kuczynski's results: First, they are based entirely on the female fertility figures. In countries in which (on account of the War or other causes) there is an excess of females, this procedure is apt to be misleading, since the low fertility of the females is due in part to depression of the marriage rate for females, a situation which contains its own remedy, as time goes on, and the numerical inequality of the sexes is automatically reduced. Second, it must be especially pointed out that the fertility of a population is probably the most elastic of all its biometric characteristics, and a relatively small change in the fertility, or a relatively small error in the registration of births, will in general produce a relatively large change or error in the natural rate of increase computed. If r is the natural rate of increase of a popula-

[1] This JOURNAL, September 1925, p. 305.

tion, and R_0 the ratio of the number of births in two successive generations, we have [1]

$$R_0 = e^{r(\alpha + \frac{1}{2}\beta r + \ldots\ldots)}. \tag{1}$$

If the fertility is now multiplied by a factor k, we have

$$kR_0 = e^{r(\alpha + \frac{1}{2}\beta r + \ldots\ldots)}. \tag{2}$$

where R_0 and α, β still denote the same constants as in (1). Differentiating we find

$$\frac{dr}{dk} = \frac{R_0 e^{-r(\alpha + \frac{1}{2}\beta r + \ldots)}}{\alpha + \beta r + \ldots}. \tag{3}$$

If the population is near the stationary state, so that r is nearly zero, and R_0 nearly unity, we have, essentially

$$\frac{dr}{dk} = \frac{1}{\alpha}$$

where α is the mean length of one generation (counted from mother to daughter). The numerical value of this is about 28.5 years, and $\frac{1}{\alpha} = .035$. Thus we have, as regards order of magnitude, $dr = .035\ dk$.

Hence if there is a change (or error by faulty birth registration) of 10 per cent in the fertility, we find $dr = .035 \times .1 = .0035$. So, for example, if r, computed from fertility and mortality, was found to be .005, the effect of the change in fertility (or the correction for incomplete registration) would make $r = .0085$.

Thus, when the rate of natural increase is itself small, a change or error in the fertility by 10 per cent produces a change or error in the natural rate of increase by 70 per cent. It is, therefore, clear that conclusions based on this mode of reckoning must be considered with caution. In any case, even if the "net reproduction" computed is true to facts, a relatively small readjustment of the fertility would remedy the defect. Will this readjustment be made when it becomes necessary? We do not know, but the recent report of a birth-rate of 19.7 per thousand for the year 1928 (as against 20.7 for the year 1927) is disconcerting. The chief danger to the civilized population lies not so much in the existing low fertility, as in the fact that, once the people at large have "discovered the convenience" of small families, there is no influence in sight that should restrain them from carrying the policy of restriction beyond the limits required for biological survival. Nature tricks man, by unreasoning instinct, to propagate. Man, through reason, discovers the trick and contrives a countertrick. He wins the move, but at the peril of losing the game.

Kuczynski's book contains a wealth of statistical material bearing on these important questions, and must be welcomed as a valuable contribution. The volume is to be followed by others, dealing with Southern and Eastern Europe, Africa, Asia, North and South America, and Australia.

ALFRED J. LOTKA

[1] A. J. Lotka, this JOURNAL, 1927, p. 163.

Ores and Industry in the Far East, by H. Foster Bain, with a preface by Edwin F. Gay. New York: Council on Foreign Relations, Inc. 1927. 229 pp.

Ores and Industry in the Far East, by H. Foster Bain, published by the Council on Foreign Relations as the first of their research reports, should dispel to a considerable degree the belief, generally current, that the Orient is a rich storehouse of untouched mineral wealth and that it is destined to have a great advance in manufacturing industries and to become a serious rival of the Western countries. It is a belief which has persisted because of the distance separating the East and the West and which has been fostered by the too enthusiastic accounts of travelers not qualified to write upon geologic and economic topics. Mr. Bain writes with full knowledge of his subject. As a mining engineer, he has had much experience in making mineral surveys, especially in China and southeastern Asia. In the preparation of the present volume, he has had access not only to the published literature but to private and unpublished reports submitted by himself and his colleagues to American mining interests. He has written an excellent survey of the mineral wealth and the industrial possibilities of the Far East with conclusions well-supported by facts.

The book is concerned only with eastern and southeastern Asia. It describes the mineral resources of China and Japan in detail, and there is some consideration of Korea, the Malay States, Indo-China, the Dutch East Indies, and the Philippine Islands. It does not include British India, Siberia, or the countries of southwestern Asia.

In the introductory chapters, the author points out the essential importance of coal and iron ore in the establishing of manufacturing industries. It is in those two minerals that the Oriental countries are especially poor. The world depends upon Asia to a large degree, at present, for antimony, tungsten, and tin, but, with the exception of the coal deposits of China, the Far East has no large deposits of coal or iron ore. Japan has small deposits of coal that will probably be insufficient for her own future needs. She has practically no iron ore. China has very rich reserves of coal, estimated at over 900 billion tons, and numerous deposits of iron ore, none of which is characterized by either large tonnage or high quality. It is upon the incorrect estimates of the iron ore of China that the many fallacious predictions regarding the industrial future of that country are based. As Tegengren, a member of the Geological Survey of China, has pointed out, "the total quantity of iron ore (both actual and potential) would be consumed by the iron industry of the United States within less than nine years." Mr. Bain is of the opinion that though China may perhaps be able to develop an iron and steel industry capable of supplying the country's needs, the Western countries need not fear any competition in world trade in those materials from this section of the Orient.

In addition to the chapters on coal and iron ore, there are chapters on sulphur, the non-ferrous metals, and the non-metallic minerals, and a chapter on petroleum contributed by W. B. Heroy, of the Sinclair Consolidated Oil corporation. In his concluding chapters, Mr. Bain quite properly directs attention to factors other than mineral resources that are important in the development of manu-

facturing industries. He points out that the supposedly cheap and abundant labor of the Orient is really neither cheap nor abundant, and that manufacturing industry must overcome many difficulties, including the conservatism of the people, their physique, the climate, the lack of capital, nepotism, and the poor transportation facilities.

In conclusion, the author emphasizes the fact that the Far Eastern countries are characterized by deficiency in mineral resources rather than by abundance. If the Asiatic countries should shift from agriculture to manufacturing—and already there is a tendency in that direction—the danger of the future will be not in any possible flood of iron and steel and manufactured goods that may come from the Orient, but in the burden that will be placed on the West in providing Asia with the essential raw materials. It is in that prospective drain upon the world's resources rather than in any yellow industrial peril that the author believes the threat to world peace lies.

JOHN E. ORCHARD

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A Scientific Approach to Investment Management, by Dwight C. Rose. New York: Harper and Brothers. 1928. 440 pp.

American Investment Trusts, by John Francis Fowler, Jr. New York: Harper and Brothers. 1928. 415 pp.

Investment Trusts, American Experience, by W. H. Steiner. New York: The Adelphi Company. 1928. 325 pp.

It used to be said that a good investment was one in which you got your money back, but Mr. Dwight C. Rose of Stevens, Scudder and Clark, Investment Counsellors, has written a book to prove just how pernicious such counsel may be. In the extremely interesting volume which results—*A Scientific Approach to Investment Management*—Mr. Rose does not even start with that traditional investment on which you got your money back. He insists that you must not only get your money back but also an annual return equal to a figure which he has computed as the riskless rental value of capital. Even such an investment, however, fails to meet the standards of a good investment—it merely avoids the stigma of being bad. To determine the qualities of a good investment, Mr. Rose turns to research and seeks to discover just how good actual investments have been. The period covered by the research extends from 1900 to 1927. In other words, his criterion of a good investment over a given period is actual investment performance. Just as life insurance companies prepare experience tables on human mortality in order to measure the risk which they undertake, Mr. Rose by painstaking and meticulous research endeavors to prepare investment experience tables which will measure the return to be expected on "good" investments. In doing this he invents various devices by which different types of investments can be compared and actual investment returns be measured. He then applies these standards to the actual investment experience of twenty-five leading fire insurance companies, and to the market record of the various stocks and bonds included in the Dow-Jones stock and bond market averages.

Naturally, the test of a good investment during this period proves to be the investment performance of successful common stocks. The period includes both the big bull market which extended from 1900 to 1907 and the bull market from 1921 to 1927. It could not help, therefore, reflecting the highly favorable experience of common stocks during these periods. Indeed, the showing would have been even more favorable to common stocks had the year 1928 been included. Since the turn of the century, accordingly, those fire insurance companies which invested heavily in common stocks made a better investment showing than those which confined themselves to bonds, and those investors who purchased the popular common stocks which the Dow-Jones Company used in computing its stock market averages realized a much more handsome return than was possible through investments in bonds. These are not altogether surprising or new conclusions. It has long been understood that speculative possibilities for profit were much larger in the stock market than in the market for bonds and that the latter offered little margin in their yield over a theoretical riskless rental rate for capital. What is surprising and new in the analysis is the extent of the advantages that lay with common stocks over this period. By means of careful analysis of individual stock records, Mr. Rose is able to show that the averages which the Dow-Jones Company published have not reflected the full appreciation in the value of the securities on which they were based and that if the effects of special dividends, rights, etc., had been included the averages would have shown a very much larger appreciation.

The comments on these results include a stimulating though not exhaustive analysis of the factors which have led to the rise in common stock values and an excellent discussion of the place of stocks in a scientific investment program. It is somewhat unfortunate that they do not also include a more comprehensive treatment of the investment merits of bonds. The research on which the study is based was evidently planned some years ago when the merits of common stocks as investments were much less appreciated than in the months surrounding its publication. It devotes only passing attention, consequently, to the specific advantages of bonds. Coming at a time of general speculative enthusiasm, the description of the results of the research, despite italicized warnings, leaves the impression of a somewhat more bullish attitude toward common stocks than was probably intended. In other words, to the extent that there is a message in the book, it is essentially one which seeks to persuade ultra-conservative investors that a sound investment program should not be confined mainly to bonds. This lesson according to all appearances had become thoroughly absorbed by the public prior to the final publication of the book.

The discussion should appeal to all who are interested in security market phenomena, especially those who seek to become acquainted with the new investment technique that is being followed by some of the leading investment trusts. Two other recent volumes should also appeal to the latter group, namely, *American Investment Trusts*, by John Francis Fowler, Jr., and *Investment Trusts, American Experience*, by W. H. Steiner. It is refreshing to find that both of these volumes differ from the older literature on investment trusts in the emphasis which they place on the organization and problems of the investment trust as

it is found in this country rather than on the experience of the older European investment trusts. These older trusts did serve as models for the original ventures in the investment trust field in this country, but the movement has developed so rapidly in recent years and along such diverse lines that the typical practice of the older European trust no longer affords an adequate background for study of the most recent innovations in the field.

WINFIELD W. RIEFLER

Principles of Valuation, by J. A. Grimes and W. H. Craigue. New York: Prentice-Hall. 1928. vii, 274 pp.

The authors are valuation engineers, in the Bureau of Internal Revenue and the Income Tax Unit of the Treasury Department respectively. The book is accordingly distinctively practical in scope and character. The basic valuation process—the practical discounting of anticipated future income—is assumed to be definitely specified. The treatment of the valuation problem thus proceeds entirely from the viewpoint of the accountant, the underlying economic-legal principles being presumed as given.

General algebraic formulas are derived for evaluating the various types of income—terminable, or perpetual; regular, or intermittent; subject to tax, or tax-free. Comparison is made between methods of valuation involving different compound interest formulas, or sinking fund formulas based on one, and more than one, interest rate.

The formulas involve only simple algebra and are worked out in profuse detail, with copious numerical examples. The primary formulas are expressed fully in charts and tables. The latter, taking up almost a third of the book, are in the main original and constitute a notable contribution to valuation technique.

As is to be expected, at several points in the discussion the authors fail to avoid controversial topics. Thus, at the beginning of Chapters II, V, and VI, views are expressed of income and interest that would be rejected by many economists.

Perhaps, also, the book is unduly restricted to elementary methods. There is no hint anywhere of even the existence of the modern refinements in mathematical valuation theory, such as are exemplified in the recent papers by Taylor, Hotelling, and Roos.[1]

MAX SASULY

Brookings Institute

Methods of Compiling Housing Statistics, Studies and Reports, Series N (Statistics), No. 13. Geneva: International Labour Office. 1928. 119 pp.

This report, prepared by the International Labour Office at the request of the International Union of Municipalities, is a welcome contribution toward the coordination of housing statistics. It should prove useful in crystallizing a definite

[1] J. S. Taylor, "Statistical Theory of Depreciation," this JOURNAL, December, 1923; H. Hotelling, "A General Mathematical Theory of Depreciation," this JOURNAL, September, 1925; C. F. Roos, "A Mathematical Theory of Depreciation and Replacement," *American Journal of Mathematics*, Vol. 50, January, 1928.

terminology and in bringing about the adoption of internationally comparable units in a field which now suffers from utter lack of uniformity. An illustration of existing conditions is found in the fact that there is no agreement on the definition of even so common a term as "dwelling." In the definition of the English census, a dwelling is "any room or set of rooms intended or used for habitation, having separate access to the street or to a common landing or staircase." Thus, each apartment in an apartment house is a separate dwelling. This definition is recommended by the International Labour Office and, in substance, has been adopted in Italy, Poland, and Germany. However, in France and Belgium the census dwelling is identical with the household and may consequently be composed even of a fraction of an apartment. On the other hand, in the census of the United States, an entire apartment house constitutes only one dwelling.

The report represents the first successful attempt to bring together the results of the experience of various municipalities and states in compiling housing statistics. Although it was prepared from an international viewpoint with a European background, it is comprehensive enough to offer valuable suggestions with respect to housing conditions in general. Those interested in this field will find in the first chapter of the study an elementary treatment of statistical procedure as applied to housing. The succeeding chapters are devoted to the discussion of definite topics pertaining to statistics of dwellings, housing conditions, rents, the housing market, and the fluctuations in total housing supply. In addition, a final section is given to conclusions, in which an excellent summary is presented, together with recommendations relative to definitions, units, and methods to be employed in the compilation of housing statistics. The report is replete with notes and references citing instances from the experience of various European states. To the American student of housing this should prove of especial interest.

Exception may perhaps be taken to the logic of some statements in the report. For instance, it is stated (page 74) that "in many countries it is the custom to concentrate the expiry of leases in a certain quarter of the year. The number of vacant dwellings will therefore vary considerably as the census is taken before, during, or after that quarter." One might well ask where the people with expired leases are housed during the quarter in question. Are there really more dwellings per family in the moving season than at any other time of the year? And if so, is it due to the expiration of leases? It is probable that this seeming lapse in reasoning is due to the inadequate definition of the term "vacant dwelling" recommended on page 75 of the report. The recommendation is made "to adopt the widest conception and to consider any habitable dwelling vacant if at the date of the census it is unoccupied and immediately available for occupancy," but a "vacant dwelling already the subject of a lease which comes into force only at a later date" is not to be counted as vacant. It is obvious that on the basis of the recommended definition, each family which has made arrangements to change its dwelling will be credited with occupying two homes at the same time—the one in which it is actually living and the one it is about to occupy. Neither of these would be counted as vacant, although it may be only a matter of hours before the first dwelling becomes available for another family.

MAURICE LEVEN

A Financial History of the American Automobile Industry, by Lawrence H. Seltzer. Boston: Houghton Mifflin Company. 1928. 297 pp.

The Automobile Industry, Its Economic and Commerical Development, by Ralph C. Epstein. Chicago: A. W. Shaw Company. 1928. 412 pp.

Nothing more clearly indicates the artificiality of the academic boundary lines among the social sciences than an attempt to classify these two books concerning the history of the automobile industry. Both are chiefly descriptive, and they supplement each other admirably in giving a broad picture of the development of our largest manufacturing industry, born such a short while ago. History, economics, sociology, psychology, and statistics can all claim them. And perhaps their greatest significance to the statistician lies in the mass of non-statistical material which they present. It is not always recognized that a great statistician, like a great historian, must continually venture into other fields of knowledge.

Professor Seltzer's study is a Hart, Schaffner and Marx prize essay. It sets as its problem: How have the leading American producers of automobiles met their capital requirements? The method is by case study; Ford and General Motors are examined in great detail, and lesser investigations are made of six other leading producers. The records are by no means limited to balance sheets and security issues; they are life-histories. The conclusion is that little capital was directly invested. An "indirect diversion of capital" took place whereby the capital employed in making cars was virtually supplied by the manufacturers of parts and accessories and by the distributors. The bulk of the present capital has come from the reinvestment of profits. While this conclusion concerning the manner in which the mobility of capital is attained is well proven for the automobile industry, the same analysis cannot necessarily be applied to other new industries where the technical or market situation may be utterly different.

The study by Professor Epstein is much less limited in its scope, and is almost impossible to summarize. It is essentially a study of business policies, and of the results of those policies. In particular, the problems of price determination and of inciting consumer demand are discussed in a most stimulating manner. But the section of most significance to statisticians is that concerning the records of the various individual concerns which go to make up the total automobile production—one of the steepest mountain sides among our many industrial trend lines. The utter impossibility of describing a "representative firm" is made clear. There is no stability among the firms in terms of rank. Even a "representative sample" is a meaningless phrase. The concerns which were most successful in the pioneering years of the industry have almost without exception fallen from the picture as maturity and stability have been approached. It is a direct challenge to the statistician who thinks of industries as made up of many like concerns, undergoing similar and parallel experiences.

Because of the paucity of data concerning profits in industry, it is perhaps important to call attention to the results of Dr. Epstein's investigation of the fortunes of leading producers since 1910. In the years before 1917, earnings averaged annually about 40 per cent of net worth; since 1917 they have averaged

about 20 per cent. Not only has the trend been downward, but the differentials among the various producers have decreased. The automobile industry is rapidly attaining its maturity.

These two books should help greatly in the intelligent use and the analysis of statistics concerning the automobile industry. It is fortunate that two such careful investigators have gathered the material so early in the life of the industry, for much information was obtained by interview or by the examination of records which may not be preserved for long. In a period when changes are as rapid as they are at present, it is certainly desirable that similar studies be made of other industries. And statistics will not be one of the lesser beneficiaries.

WILLARD L. THORP

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Die Neue Goldwaehrung. (Weltwirtschaftliche Zusammenarbeit), by Arthur Schweizer, Mit einem Vorwort by G. Bachmann, Praesident des Direktoriums der Schweizerischen Nationalbank. Basle, Switzerland: Verlag Helbing & Lichtenhahn. 1929. viii, 216 pp.

Schweizer's treatise on the *New Gold Currency* appears at a time when most of the civilized countries have returned to the gold standard currency, but also when an organization for solving questions of international concern arising out of currency problems is only about to start. Schweizer begins his book with the philosophical question as to whether trade revival or the restoration of the gold currency should first be taken care of, and apparently decides in favor of the primacy of currency restoration. The title of the book, *New Gold Currency*, indicates that the problem of returning to the gold standard currency in European countries involved not simply the application of the old rules to current circumstances, but required the creation of new systems on the foundation of partly new knowledge. The interdependence of the currencies the world over had become more evident after the World War than ever before.

After a review of the principal theories of currency, the author, although aware of some of its clearly demonstrated weaknesses, regards with favor a currency on the gold standard basis on account of its alleged practical superiority. He then surveys recent currency history in various countries, and a good choice of statistical data helps to complete the otherwise necessarily compressed description. The third and fourth chapters of the book may be regarded as of special interest to the American reader. They are entitled "Gold" and "The Policy of the Centralbanks especially from the aspect of international economic coöperation." In "Gold" Schweizer weighs the objections to gold as a currency factor brought forth in a number of treatises especially by Gustav Cassel, John Maynard Keynes, Irving Fisher and others, and lays stress upon the decided influence of gold movements and Centralbank policy on the *stability* of the value of gold, the latter being "the main interest of the whole commercial world." Schweizer comes to the conclusion that comparatively low gold production, and on the other hand a strong gold demand will create an undesirable increase in the value of gold, which has already partly been offset and still has to be checked by gold movements. He

upholds a fairly general European view that the United States of America possesses gold much in excess of its need. Without intention of giving this point more attention than it receives in Schweizer's book itself, we believe that percentage comparison alone with the pre-war years is not sufficient to support this argument. Schweizer realizes the fact, too, that a new important gold market has again been created in Europe and he expects a satisfactory distribution of the available gold resources from coöperation. This thought leads to his last chapter in which the latest achievements towards organising common interests and reaching international understanding in Centralbank policy are discussed. His subtitles are: (1) The thought of world-wide economic coöperation; (2) gold and money market policy; (3) discount rate and credit policy; (4) form of coöperation by Central Banking Institutions. He rejects F. H. Vanderlip's plan for a Pan European Centralbank, for the present European status in currency, politics and economics does not call for unification of European States in regard to a common currency, but he wants a world wide understanding between all money-issuing institutions, at the same time preserving national individuality. He pays tribute to the late Governor Benjamin Strong, who played a leading rôle in promoting and stimulating international coöperation. It is a distinction of this treatise that a broad aspect of the complex subject is kept up throughout in brief and clear form supported by a rich literary allusion. It suffers, perhaps, from a certain lack of exact paraphrasing in more or less important details. The well-arranged book is dedicated to Leopold Dubois and contains an introduction by G. Bachmann, President of the Board of Directors of the Swiss National Bank. Schweizer himself is connected with the Swiss Bank Corporation in Basle, Switzerland.

RUDOLF BAUMGARTNER

Bankers' Balances: A Study of the Effects of the Federal Reserve System on Banking Relationships, by Leonard L. Watkins. Chicago: A. W. Shaw Company. 1929. 429 pp.

One of the major aims of the Federal Reserve Act was the ultimate centralization of bank reserves in the hands of the regional institutions so that bankers' balances would be separated from reserve funds and reduced to an innocuous position. It was considered almost a certainty by many proponents of the Act that inter-bank deposits would shrink in amount as they declined in importance. In this volume, which is the first to win the triennial prize offered by the Chicago Trust Company, Professor Watkins undertakes to evaluate the changes which have been brought about in the size and functions of bankers' balances by the establishment of the Federal Reserve system. It is not easy reading for every statement is carefully bulwarked by statistical data. But the volume well repays careful study, and there are parts of it which the specialist in banking problems will wish to read more than once. The motives which led bankers, before the organization of the Federal Reserve Banks, to maintain inter-bank deposits, are grouped into four categories. In an elaborate analysis, based largely upon a comparison of banking statistics for the two thirteen-year periods immediately

preceding and immediately following the organization of the new banks, the influence of the Federal Reserve system on each of the four factors is considered.

The most important of these factors before 1914, was the desire to maintain reserves in what was at once a legal and a profitable form. Since that date, the changes in reserve requirements have eliminated this particular motive in the case of member banks. For many banks outside of the system, however, deposits with other banks may still be counted as part of the required legal reserve, and to this fact may be ascribed a large part of the "Due from Banks" appearing on non-member bank balance sheets. To a certain extent, country member banks still look upon their deposits with city banks as a form of secondary reserve, even though they may not be counted as part of the legal minimum reserve required by the Federal Reserve Act. While the reserve motive in bankers' balances has been heavily undermined by the Federal Reserve Banks, therefore, it has not been entirely eliminated.

Another strong incentive to the maintenance of balances with correspondent banks before 1914, was the service customarily rendered by the correspondent in the clearing and collection of checks. In spite of the mechanism which has been developed by the Federal Reserve Banks for the same purpose, this service is still considered a valuable one by country and non-member banks. Even city member banks sometimes, from habit or policy, prefer collecting in the old way through correspondents, to sending the checks through the Federal Reserve channels. Although it has lost much of its force since the legislation of 1914, the collection service is still an incentive to the keeping of bankers' balances.

In the period before 1914, when there was no central bank to rediscount commercial paper, the assurance that its city correspondent would permit it to borrow in case of need, was a third factor in influencing country banks to maintain balances in larger amount than would otherwise have been the case. The Federal Reserve Banks have supplemented rather than supplanted this type of service. For non-member banks, and for banks within the system which hold ineligible paper, borrowing from a city correspondent is a frequent necessity, and the total amount of inter-bank loans is now far above anything witnessed under the old system. The privilege of borrowing is still, therefore, an important element in bankers' balances.

Although Professor Watkins does not over-emphasize the payment of interest on balances as a factor in determining the share of bank funds kept on deposit with another bank, he believes that this "investment factor" was always present. Since the establishment of the Federal Reserve system, the small country bank is the only one which makes use of this form of investment for its funds to any great extent. Other banks tend to treat their inter-bank deposits as "avenues of investment" through which purchases of securities can be made, rather than as an end in themselves. The proportion of bankers' balances to the total loans and investments of all banks has, therefore, fallen since 1914. The growth of the acceptance market, the newly-opened possibility of making real estate loans, and particularly the attractiveness of the call money market, have seriously weakened the position of bankers' balances as a form of investment for surplus bank funds.

Bankers' balances, although they have increased in amount since 1914, have de-

clined relatively to the growth of other bank assets. Much of this change can be ascribed to the effect of the Federal Reserve Act, which has weakened the force of each factor leading to inter-bank deposits. But this proportionate shrinkage had been under way, at a slower rate, even before 1914, and it is evident that there were elements in the banking system which would have brought about some decline in the importance of bankers' balances had the Federal Reserve system never been established.

On the larger questions of Federal Reserve policy which are bound to arise in such a discussion, this volume throws many interesting sidelights. New York City is still the focal point for bankers' balances. But since the Federal Reserve Act has accomplished its purpose in severing the dangerous connection between bankers' balances and bank reserves, Professor Watkins concludes that it is unnecessary, and would probably be undesirable, for the system to insist upon absorbing all of the services now performed by correspondent banks in order still further to reduce the balances. There is no reason for the Reserve Banks to take over what remains of the clearing and collection of checks; nor should they assume the burden, now carried by correspondent member banks, of investing for banks. In the opinion of the author, therefore, the problem of bankers' balances has been practically solved, and the Reserve Banks should direct their attention to the problem of call loans. Such loans were formerly closely related to bankers' balances, but this connection is no longer so close; while bankers' balances have been declining in proportion to total investments of member banks, call loans have been increasing. Call loans have assumed the rôle formerly played by bankers' balances, and are the pressing banking problem of the day.

MARGARET G. MYERS

Overhead Costs and the Shift to Machinery, by D. D. Kennedy. Philadelphia: University of Pennsylvania. 1928. 100 pp.

This monograph rests, so far as quantitative evidence is concerned, chiefly on the records of only ten plants; and hence, as the author recognizes, is necessarily illustrative of the problem and methods of analysis rather than conclusive as to precise relations.

As illustrated by the sample data, overhead expenses, defined as indirect expenses chargeable against the factory department, increase in importance with the shift to machinery; for both machine and indirect labor costs, which are relatively non-variable, increase in proportion to total expense.

As a basis for classifying expenses as variable, semi-variable, and non-variable, coefficients of correlation are computed for month-to-month fluctuations between machine hours worked, taken as a measure of productive activity, and each class of expense.

The importance of such overhead costs is sharply accentuated by non-use of capacity, whether it be due to shortened standard working time, idle capacity within the standard working period, or off-standard production even when equipment is in use. Hence the increasing importance of overhead costs involves an increasing financial inducement for regularization of employment.

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With the general conclusions reached by Mr. Kennedy, there will probably not be much dissension. It is obvious, however, that the precise weight to be attached to the evidence he submits is qualified by the smallness of the sample and by some unavoidable uncertainty as to whether the accounting practices followed in determining standard production and distributing costs are uniform and conformable to the technical and economic realities of factory operations.

The propriety of the author's use of the terms "coefficient of variation" and "coefficients of correlation" as interchangeable (pages 48–49) may be questioned; and occasionally there has been a lack of care in clearly and accurately identifying the data used (see captions of Table XXIV); but in general the explanation is clearly put and well reasoned. This monograph, despite its limited basis, is a worth-while contribution in helping to blaze the way for much-needed intensive studies of the causes and significance of increasing productivity and of the effects of increasing mechanization, particularly with reference to their bearing on the insecurity of employment.

HARRY JEROME

Casualty Insurance, by C. A. Kulp. New York: Ronald Press. 1928. vii, 610 pp.

Professor Kulp's text on *Casualty Insurance* fills admirably a gap in insurance text literature. With the publication of this book there are now standard texts on each of the broad fields of insurance; life, fire, marine and casualty.

While this is presumably a general text, nearly one-half of the book is devoted to employers' liability and workmen's compensation insurance. It is urged, in justification of this allocation of space, that casualty insurance has been most highly developed in those branches, and that an explanation of practices there may be used as a basis for an understanding of practices in other fields. The remainder of the book is devoted to a detailed examination of coverages and rates in the principal remaining fields of casualty insurance. A selected bibliography is appended.

The reviewer has two principal adverse criticisms: (1) what seems to him to be an undue emphasis on workmen's compensation insurance in a general text on casualty insurance, and (2) the extreme development of detail in the analysis of coverages and rates for other types of insurance. The first criticism explains itself. The second can best be elucidated by stating that once again the trees seem to have, in some cases, obscured the forest. The author has been at such pains to be complete that a reading of his chapter on any particular kind of insurance leaves a confused impression of infinite detail, rather than a well-rounded understanding of the subject of the chapter. The reviewer would not emphasize too greatly these two criticisms. The material on workmen's compensation is authoritative and well organized, and constitutes an excellent text in itself on that subject. While chapters on other forms of insurance may be at times rather difficult reading, the interested student will find brought together for the first time an adequate explanation of principles and practices in the casualty insurance field.

The book is to be recommended particularly to those who are already interested in this field, and as a text for insurance classes.

RALPH H. BLANCHARD

Columbia University

The California and Hawaiian Sugar Refining Corporation, by Bores Emmett. Stanford University Business Series No. II. 1928. 294 pp.

As the name, *The California and Hawaiian Sugar Refining Corporation*, suggests, this volume is a descriptive and analytical study of the California refining plant of the Hawaiian sugar growers. A study of this company is of particular interest because of the coöperative nature of the enterprise and the scope of its activities, concerned as they are with the production of raw material, transportation, manufacture, distribution and finance. Furthermore, the narrow margin on which the sugar refiner operates requires an unusually exact control of operations and costs by the management. Hence, the analysis of the methods of control applied by this company furnishes the student or business man with an exceptionally informative example of modern management principles expertly applied. Thus, this book takes its place in that fast growing field of business literature which illustrates and thereby teaches by concrete examples, the principles of management common to all enterprise.

The initial sections describe, with the aid of tables and charts, the operations and problems of the business, covering in detail the refining, sales, finance and accounting divisions and the legal department. Contained in the chapter on sales, is an explanation of a simple method for correcting sales figures for normal seasonal variation, which many will find useful. The final section on progress and results is perhaps the most interesting to the statistician because it illustrates a variety of ways through which achievement is statistically measured as a practical aid to executive control.

THURSTON P. BLODGETT

La Science Economique! Ses lois inductives, by Yves Guyot. Paris: Alfred Costes. 1928. xxii, 405 pp.

This volume is announced on the cover as "sixth edition, entirely reworked." The well-known treatise of M. Guyot was published in 1881 as a volume of 470 pages. It was rewritten for the third edition of 1907, in 531 pages. An insert sheet from the publishers explains that the exposition of the perturbations due to the Great War necessitated expansion of the work into two volumes, of which the first was published on the very day of the author's death, and may be regarded as his economic testament. The inference is that the revision of the second half was not carried far enough to warrant giving it to the public.

The first six books (out of a total of ten) in the old edition have become fourteen, aggregating sixty-three chapters. Hence there are only about five actual pages of text to the chapter. This organization indicates the neat, epigrammatic way in which M. Guyot disposes of economic questions, one after another. The fundamentals of economics are dealt with in four books, of which the second dis-

poses of the subject of statistics in one chapter of twenty pages. The expression *vérité économique* recurs constantly. Monetary theory and history occupy eight books. The last is a "unique chapter" announcing in ten pages that Volume II will deal with the conflict between the old civilization and the new; between the civilization of seizure and of faith in supernatural fetiches like society and the state—in a word, military and sacerdotal civilization—and the new civilization of science, of production and exchange, or economic civilization.

The book is essentially a succession of sermonic attacks and exhortations, quite in the "Manchester" vein, against interference in all forms and in favor of stalwart individualism. The peculiar character of nineteenth-century French economics, in relation to the character of the economic life of the country, might be the topic of an interesting study in historical interpretation, but it cannot be written, or begun, here. The expansion in the text has taken place chiefly in the discussion of the monetary experiences of the different countries during and after the war. The narrative is interesting, but the author's whole interest and emphasis center on drawing the moral, which is to beware of another of his favorite monsters, inflationism. This of course is well enough. But no one should be misled by the subtitle. Induction to M. Guyot means simply the correct interpretation of ordinary experience and observation, or in other words, whatever he regards as obviously true.

FRANK H. KNIGHT

University of Chicago

A Way of Order for Bituminous Coal. By Walton H. Hamilton and Helen R. Wright. (Institute of Economics Series.) New York: The Macmillan Company. 1928. ix, 365 pp.

This book has nothing to offer to statisticians as such. It takes as proven the indictment against the industry which the same authors drew up, with ample quantitative evidence, in *The Case of Bituminous Coal*, and proceeds directly to the task of devising "a way of order" to replace the present disorder. The job is one of social invention and not of factual analysis. It deals with problems in which, as it points out, "the incommensurable has not yet taken on incommensurability." The result is the phenomenon of a full-length book on coal without a single column of figures.

The volume, nevertheless, has much to offer to statisticians who chance also to be concerned with economic theory and industrial policy. Its problem is defined in terms of the ends which the industry should serve and the interests which it should promote. The main functions are two: To provide coal for those who need it, and to provide a life for those who must work in it. These interests, moreover, those of the consumers and the mine workers, are the only two that can neither be commuted nor bought off. This initial analysis leads in the end directly to the somewhat astonishing solution which the authors propose. If these are the chief interests involved, why not put their spokesmen directly and explicitly in charge of an industry so clearly in need of central guidance? To this end the authors propose that present owners be paid off by an issue of debentures,

and that—as Henry Clay first suggested to the British Sankey Commission—complete authority over the industry should be vested in a corporation in which half of the control should be held by directors elected by the miners and the other half by direct representatives of the consumers. "Here the interests of producers are to be served by producers; those of consumers by consumers. The two interests are balanced and each is checked by the other; neither can be served unless the two parties work together."

This conclusion is not reached, however, until the various rival plans have first been analyzed in terms of the two objectives and in terms of the pressing need of some sort of unified control over productive capacity. Minor "patches" such as those advocated by the Coal Commission are thrown aside as inadequate. The suggestions of complete unionization, of complete trustification, and of regulation by a powerful government commission are examined with greater care. Each is found to offer an important element to be worked into a complete system, but each either leaves vital interests unguarded or else leads to an undue division of authority. A brief review can hardly do justice to the skill with which these alternatives are explored or to the persuasive though over-luxuriant dialectic with which the authors defend their own proposals. The case against "nationalization" is treated less fully. It appears to rest largely upon the more direct representation of consumers in the "Clay Scheme" and perhaps also upon the fact that the "big business" type of executive comes back into the directorate as the representative of the great industrial purchasers of coal. "One searches almost in vain for another industry in which the consumers are a compact body of industrial concerns easily organized." If this makes it impossible to regard the proposal as a "pattern" for other industries, it is perhaps the shrewd advantage taken of this exceptional circumstance that may most recommend it when the time is ripe for the serious debate of plans of drastic reorganization.

But when is such a time likely to come? The authors take little responsibility for the answer. They once defined their task as that of devising a scheme that would be "practical" rather than "feasible," the former meaning one that would work if it were tried and the latter one that people could be persuaded to try. Very likely they have chosen the better part, but their acceptance of this limitation must make both the matter and the manner of the book rather exasperating to those who are trying to "do something about it" here and now in a ruling atmosphere of such complete indifference that by contrast even the proposals of the Coal Commission wear an air of revolutionary boldness. Yet those who hope for a freer spirit of industrial experimentation will find few weapons more serviceable to their hands than the authors' analysis of "The Requisites of Order" or their development of the assumption that an industry can be judged as a unit which is or is not giving reasonable performance. And if present-day officials and present-day reformers do not quite know what to do with the book, let them file it away with the instructions: "Bring forward fifty years from date."

CARTER GOODRICH

University of Michigan

RECENT LITERATURE

DUE to the appearance in March of the new journal

SOCIAL SCIENCE ABSTRACTS

which is designed to cover fully and for the several social sciences the field dealt with superficially in the Recent Literature section of our JOURNAL, this section has been discontinued.

Social Science Abstracts is published monthly and will afford thorough abstracts of new material appearing in periodicals, government publications, bulletins, reviews, etc. It covers foreign as well as local sources in the fields of cultural anthropology, economics, history, human geography, political science, sociology and statistics.

For further information the reader is referred to our JOURNAL, June, 1928, Vol. 23, No. 162, p. 187, and December, 1928, Vol. 23, No. 164, p. 448, and to

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6